

## MOSFET Digital Circuits

### NMOS Inverter

## MOSFET Digital Circuits

- In the late 70s as the era of LSI and VLSI began, **NMOS** became the fabrication technology of choice.
- Later the design flexibility and other advantages of the CMOS were realized, **CMOS** technology then replaced NMOS at all level of integration.
- The small transistor size and low power dissipation of CMOS circuits, demonstration principal advantages of CMOS over NMOS circuits.

## NMOS Inverter

- For any IC technology used in digital circuit design, the basic circuit element is the **logic inverter**.
- Once the operation and characterization of an inverter circuits are thoroughly understood, the results can be extended to the **design of the logic gates and other more complex circuits**.

## MOSFET Digital Circuits

NMOS logic circuits

CMOS logic circuits  
complementary MOS

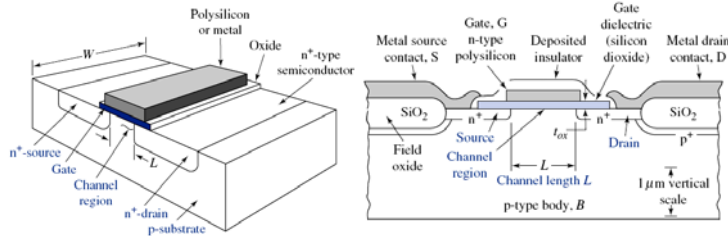
### NMOS INVERTERS

MOS inverter

NOR and NAND gates

The inverter is the basic circuit of most MOS logic circuits.

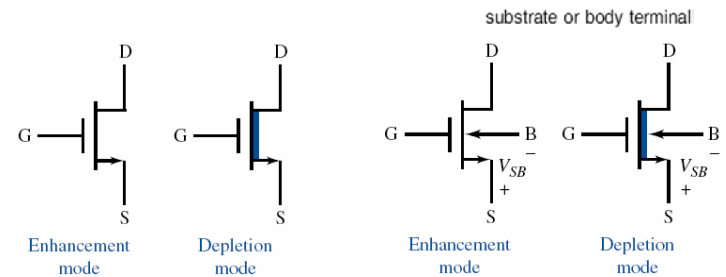
## n-channel MOSFET



n-channel MOSFET

the channel length is the same for all transistors, while the channel width is variable.

## n-channel MOSFET



n-channel MOSFET

## Chap.3

### n-Channel MOSFET Formulas



#### Transition points

$$v_{DS}(\text{sat}) = v_{GS} - V_{TN}$$

$V_{TN}$  n-channel threshold voltage

#### Saturation region $v_{DS} > v_{DS}(\text{sat})$ $v_{GS} > V_{TN}$

$$i_D = K_n(v_{GS} - V_{TN})^2$$

#### Nonsaturation region $v_{DS} < v_{DS}(\text{sat})$

$$i_D = K_n[2(v_{GS} - V_{TN})v_{DS} - v_{DS}^2]$$

$$K_n = \frac{W\mu_n C_{ox}}{2L} \quad \text{conduction parameter}$$

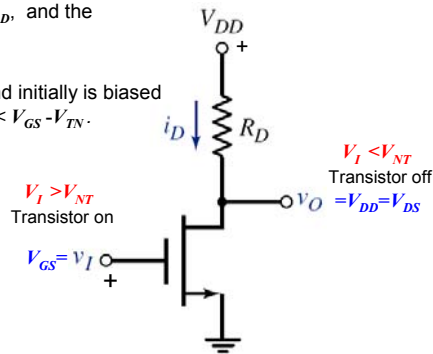
## NMOS Inverter

- For any IC technology used in digital circuit design, the basic circuit element is the **logic inverter**.
- Once the operation and characterization of an inverter circuits are thoroughly understood, the results can be extended to the **design of the logic gates and other more complex circuits**.

## NMOS Inverter

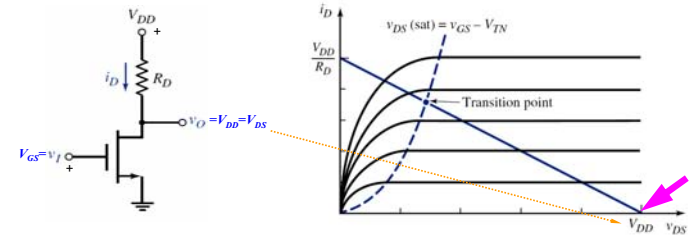
➤ If  $V_I < V_{TN}$ , the transistor is in **cutoff** and  $i_D = 0$ , there is no voltage drop across  $R_D$ , and the output voltage is  $V_o = V_{DD} = V_{DS}$

➤ If  $V_I > V_{TN}$ , the transistor is **on** and initially is biased in **saturation region**, since  $V_{DS} < V_{GS} - V_{TN}$ .



➤ As the input voltage **increases** ( $V_{GS}$ ), the drain to source voltage ( $V_{DS}$ ) **decreases** and the transistor inter into the **nonsaturation region**.

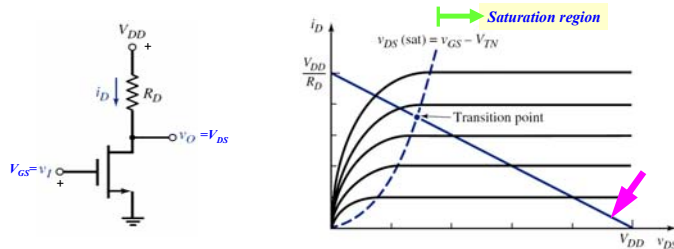
## NMOS Inverter with Resistor Load



**Cut-off**

➤ If  $V_I < V_{TN}$ , the transistor is in **cutoff** and  $i_D = 0$ , there is no voltage drop across  $R_D$ , and the output voltage is  $V_o = V_{DD} = V_{DS}$

## NMOS Inverter with Resistor Load

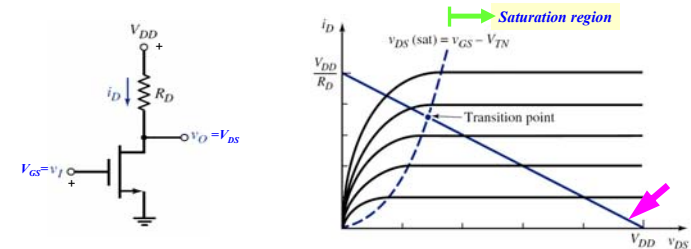


**Saturation Region**

➤ As the **input** is increased slightly above the  $V_{TN}$ , the transistor **turns on** and is in the **saturation region**.

$$\begin{aligned} v_O &= V_{DD} - i_D R_D \\ i_D &= K_n (v_{GS} - V_{TN})^2 = K_n (v_I - V_{TN})^2 \end{aligned} \quad \left. \vphantom{\begin{aligned} v_O &= V_{DD} - i_D R_D \\ i_D &= K_n (v_{GS} - V_{TN})^2 = K_n (v_I - V_{TN})^2 \end{aligned}} \right\} v_O = V_{DD} - K_n R_D (v_I - V_{TN})^2$$

## NMOS Inverter with Resistor Load



**Transition Region**

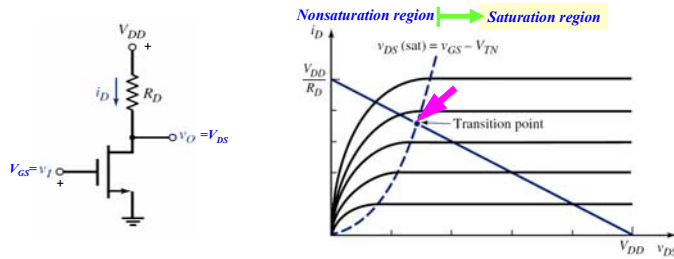
➤ As the **input** voltage is further increases and voltage drop across the  $R_D$  become sufficient to reduce the  $V_{DS}$  such that  $V_{DS} \leq V_{GS} - V_{TN}$   $v_{DS}(\text{sat}) = v_{GS} - V_{TN}$

➤ The **Q**-point of the transistor moves up the load line.

$$\begin{aligned} v_O &= V_{DD} - K_n R_D (v_I - V_{TN})^2 \\ \text{At the transition point, } V_{O1} &= V_{I1} - V_{TN} \end{aligned} \quad \left. \vphantom{\begin{aligned} v_O &= V_{DD} - K_n R_D (v_I - V_{TN})^2 \\ \text{At the transition point, } V_{O1} &= V_{I1} - V_{TN} \end{aligned}} \right\} \begin{aligned} V_{O1} & \text{ drain-to-source voltage} \\ V_{I1} & \text{ gate-to-source voltage} \end{aligned}$$

$$K_n R_D (V_{I1} - V_{TN})^2 + (V_{I1} - V_{TN}) - V_{DD} = 0$$

## NMOS Inverter with Resister Load



### Nonsaturation Region

- As the **input** voltage becomes greater than  $V_H$ , the  $Q$ -point continues to move up the load line, and the transistor becomes biased in the nonsaturation region.

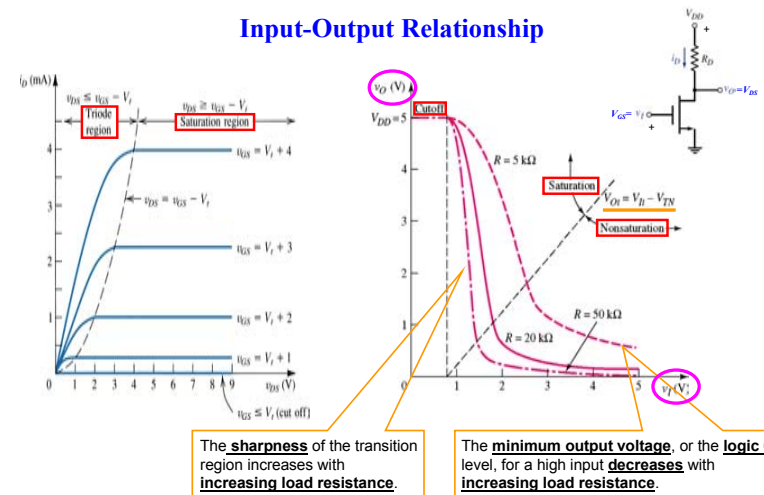
$$i_D = K_n [2(v_{GS} - V_{TN})v_{DS} - v_{DS}^2] = K_n [2(v_I - v_{TN})v_O - v_O^2]$$

$$v_O = V_{DD} - i_D R_D$$

$$v_O = V_{DD} - K_n R_D [2(v_I - V_{TN})v_O - v_O^2]$$

## NMOS Inverter with Resister Load

### Input-Output Relationship



The **sharpness** of the transition region increases with **increasing load resistance**.

The **minimum output voltage**, or the **logic 0** level, for a high input **decreases** with **increasing load resistance**.

## Summary of NMOS inverter with Resister Load \*

### Current-Voltage Relationship

**Saturation Region**  $i_D = K_n (v_{GS} - V_{TN})^2 = K_n (v_I - V_{TN})^2$

$$v_O = V_{DD} - K_n R_D (v_I - V_{TN})^2$$

**Transition Region**  $V_{O1} = V_H - V_{TN}$

$$K_n R_D (V_H - V_{TN})^2 + (V_H - V_{TN}) - V_{DD} = 0$$

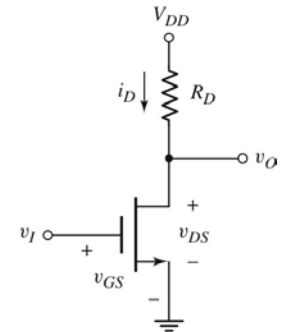
**Nonsaturation Region**

$$i_D = K_n [2(v_I - v_{TN})v_O - v_O^2]$$

$$v_O = V_{DD} - K_n R_D [2(v_I - V_{TN})v_O - v_O^2]$$

## Example

For the NMOS inverter shown in Fig.  $V_{DD} = 3\text{ V}$ . Assume transistor parameters of  $K'_n = 60 \mu\text{A}/\text{V}^2$ ,  $W/L = 5$ , and  $V_{TN} = 0.5\text{ V}$ . (a) Find the value of  $R_D$  such that  $v_o = 0.1\text{ V}$  when  $v_I = 3\text{ V}$ . (b) Using the results of part (a) determine the transition point for the driver transistor



### EX16.2

(a)

$$v_o = V_{DD} - I_D R_D$$

$$v_o = 3 - \left(\frac{0.06}{2}\right) \left(\frac{W}{L}\right) [2(3-0.5)v_o - v_o^2] R_D$$

$$v_o = 0.1$$

$$0.1 = 3 - \left(\frac{0.06}{2}\right) (5) [(5)(0.1) - (0.1)^2] R_D$$

$$0.1 = 3 - 0.0735 R_D$$

$$R_D = 39.5 \text{ K}$$

(b)

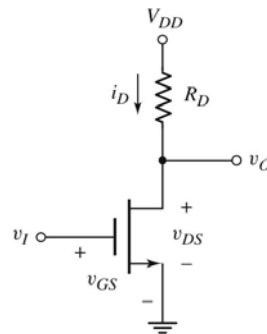
$$\left(\frac{0.06}{2}\right) (5) (39.5) (V_h - 0.5)^2 + (V_h - 0.5) - 3 = 0$$

$$5.925 (V_h - 0.5)^2 + (V_h - 0.5) - 3 = 0$$

$$(V_h - 0.5) = V_{Ot} = \frac{-1 \pm \sqrt{1 + 4(5.925)(3)}}{2(5.925)}$$

$$V_{Ot} = 0.632 \text{ V}$$

$$V_h = 1.132 \text{ V}$$



## n-Channel MOSFET connected as saturated load device

- An n-channel enhancement-mode MOSFET with the gate connected to the drain can be used as load device in an NMOS inverter.
- Since the gate and drain of the transistor are connected, we have  $V_{GS} = V_{DS}$ .

When  $V_{GS} = V_{DS} > V_{TN}$ , a non zero drain current is induced in the transistor and thus the transistor operates in **saturation only**. And following condition is satisfied.

$$V_{DS} > (V_{GS} - V_{TN})$$

$$V_{DS}(\text{sat}) = (V_{GS} - V_{TN}) \text{ because } V_{GS} = V_{DS} \text{ or } V_{DS}(\text{sat}) = (V_{GS} - V_{TN})$$

In the saturation region the drain current is  $i_D = K_n (V_{GS} - V_{TN})^2 = K_n (V_{DS} - V_{TN})^2$

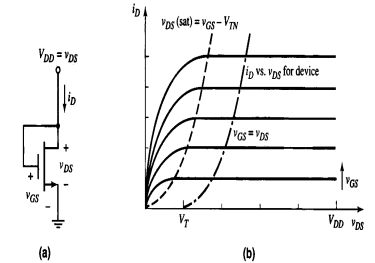
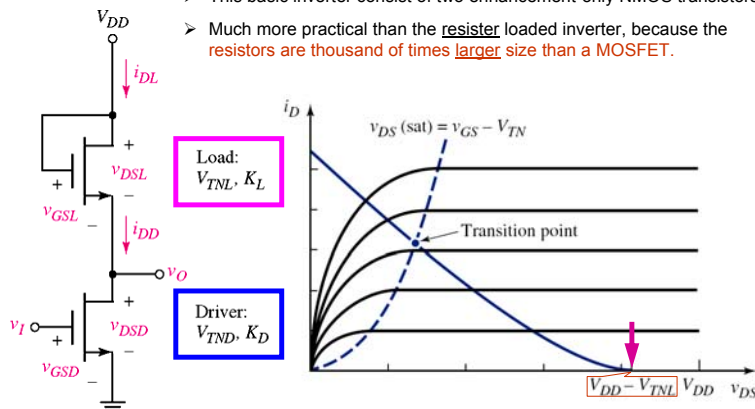


Figure 16.7 (a) n-channel MOSFET connected as saturated load device and (b) current-voltage characteristics of saturated load device

The  $i_D$  versus  $v_{DS}$  characteristics are shown in Figure 16.7(b), which indicates that this device acts as a nonlinear resistor.

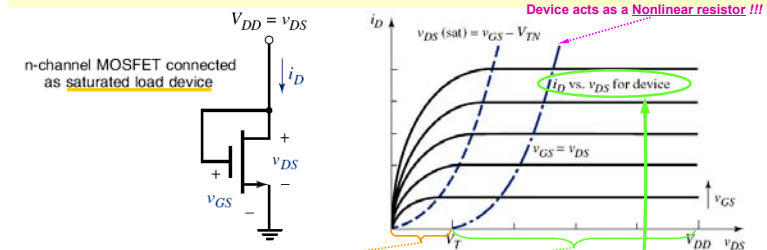
## NMOS Inverter with Enhancement Load

- This basic inverter consist of two enhancement-only NMOS transistors
- Much more practical than the resistor loaded inverter, because the resistors are thousand of times larger size than a MOSFET.



## NMOS Inverter with Enhancement Load

- An n-channel **enhancement-mode MOSFET** with **gate connected to the drain** can be used as a **load device**.



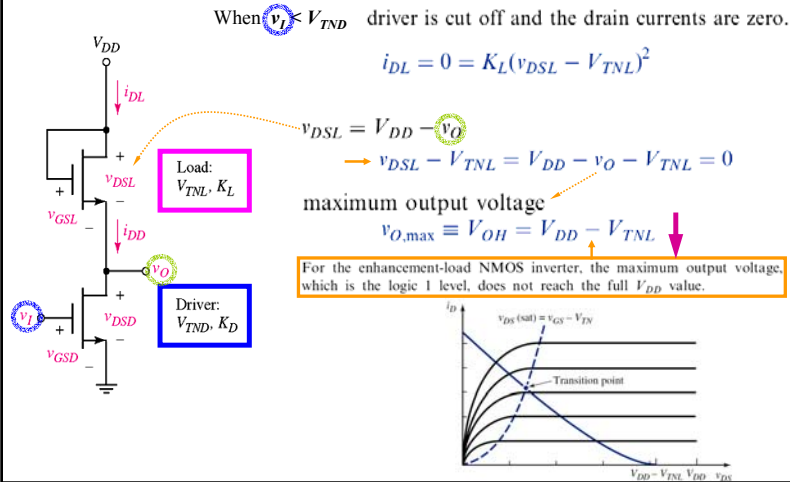
For  $v_{GS} = v_{DS} \leq V_{TN}$ , the drain current is zero.  
 $v_{GS} = v_{DS} > V_{TN}$ , a nonzero drain current is induced in the device.

$$v_{DS} > (v_{GS} - V_{TN}) = (v_{DS} - V_{TN}) = v_{DS}(\text{sat})$$

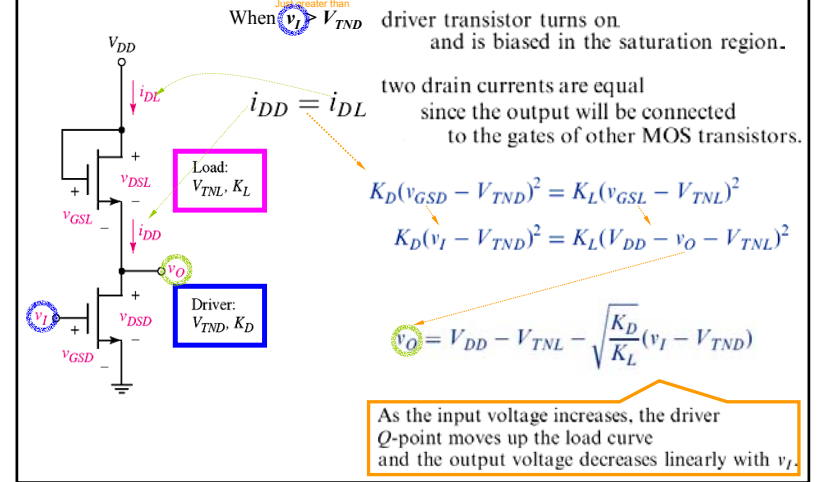
$$i_D = K_n (v_{GS} - V_{TN})^2 = K_n (v_{DS} - V_{TN})^2 \text{ Nonlinear resistor !!!}$$

A transistor with this connection always operates in the saturation region when not in cutoff.

## NMOS Inverter with Enhancement Load



## NMOS Inverter with Enhancement Load



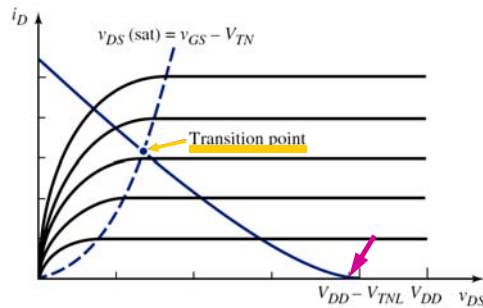
## NMOS Inverter with Enhancement Load

At the driver transition point,

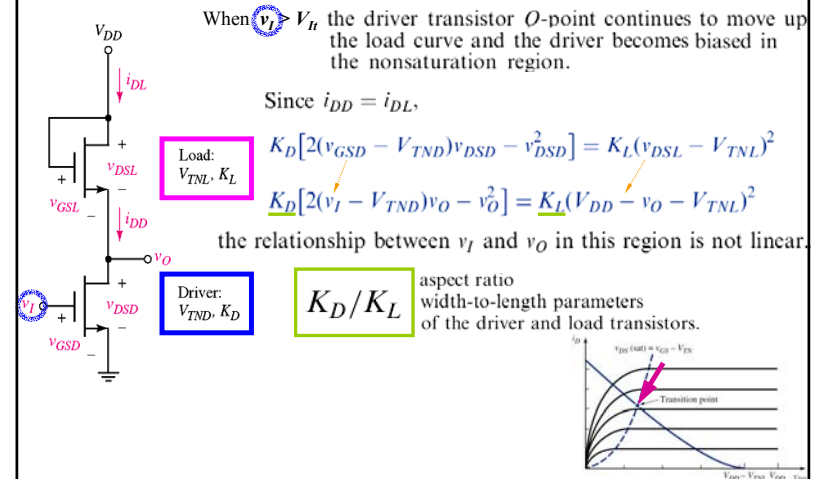
$$v_{DSD}(sat) = v_{GSD} - V_{TND} \quad \text{or} \quad V_{Oi} = v_{Ii} - V_{TND}$$

$$V_{Ii} = \frac{V_{DD} - V_{TNL} + V_{TND} \left(1 + \sqrt{\frac{K_D}{K_L}}\right)}{1 + \sqrt{\frac{K_D}{K_L}}}$$

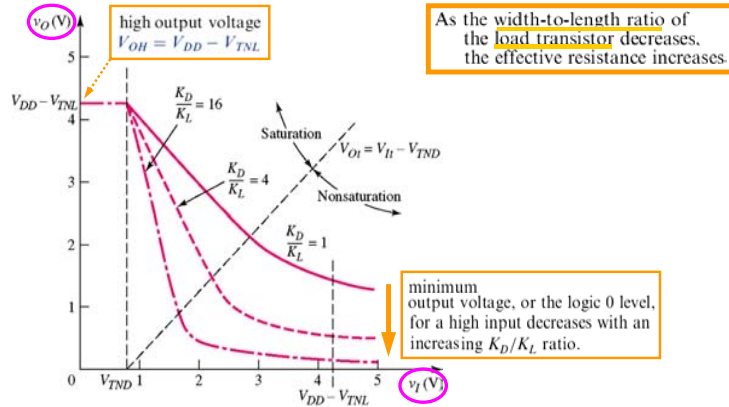
$$v_O = V_{DD} - V_{TNL} - \sqrt{\frac{K_D}{K_L}}(v_I - V_{TND})$$



## NMOS Inverter with Enhancement Load



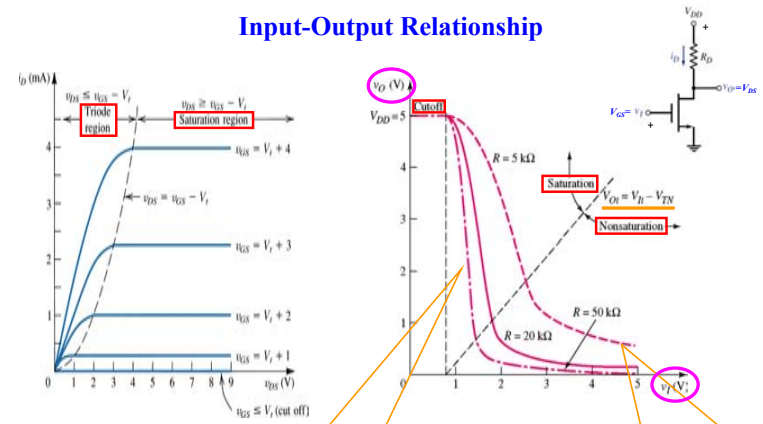
## NMOS Inverter with Enhancement Load



Voltage transfer characteristics, NMOS inverter with saturated load, for three aspect ratios

## c.f. NMOS Inverter with Resistor Load

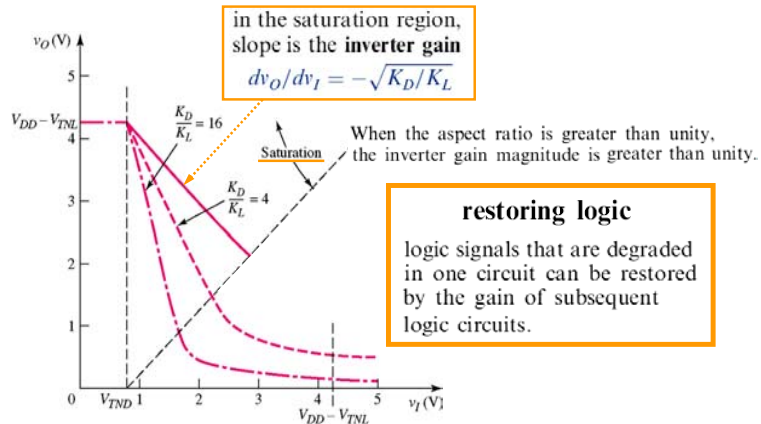
### Input-Output Relationship



The **sharpness** of the transition region increases with **increasing load resistance**.

The **minimum output voltage**, or the **logic 0** level, for a high input **decreases** with **increasing load resistance**.

## NMOS Inverter with Enhancement Load



Voltage transfer characteristics, NMOS inverter with saturated load, for three aspect ratios

P1014

### Example

**Objective:** Design the aspect ratio  $K_D/K_L$  to produce a specified low output voltage, and determine the power dissipation in the inverter with enhancement load for a minimum  $W/L$  ratio for the load transistor. (Neglect the body effect.)

Consider the inverter shown in Figure 16.8(a) biased at  $V_{DD} = 5 \text{ V}$ . The transistor parameters are:  $V_{TND} = V_{TNL} = 0.8 \text{ V}$  and  $k_n' = 35 \mu\text{A/V}^2$ . Determine  $K_D/K_L$  such that  $v_O = 0.10 \text{ V}$  when  $v_I = \text{Logic 1} = 4.2 \text{ V}$ , and determine  $(W/L)_D$  and the power dissipation in the inverter for  $(W/L)_L = 0.5$  and  $v_I = 4.2 \text{ V}$ .

### Limitation of Enhancement Load inverter



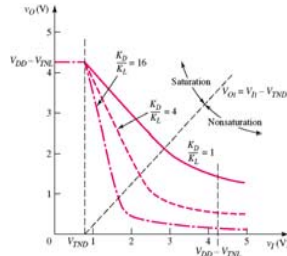
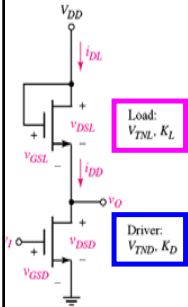
### Limitation of Enhancement Load inverter

For  $v_I = 4.2$  V, the driver transistor is in the nonsaturation region.

$$K_D[2(v_I - V_{TND})v_O - v_O^2] = K_L(V_{DD} - v_O - V_{TNL})^2$$

$$\frac{K_D}{K_L}[2(4.2 - 0.8)(0.1) - (0.1)^2] = (5 - 0.1 - 0.8)^2 = 25.1$$

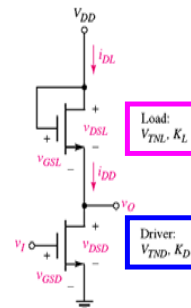
$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} \quad (W/L)_D = 12.6 \quad \text{when } (W/L)_L = 0.5$$



### Example 16.3

PI014

The power dissipated in the inverter is  $P = i_D V_{DD} = (147)(5) = 735 \mu\text{W}$



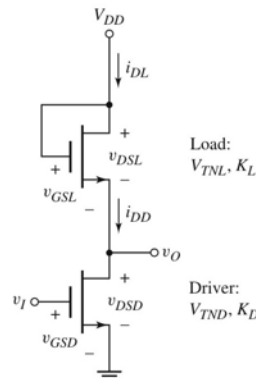
from the load transistor,

$$i_D = K_L(V_{DD} - v_O - V_{TNL})^2 = \frac{k'_n}{2} \left( \frac{W}{L} \right)_L (V_{DD} - v_O - V_{TNL})^2 = \left( \frac{35}{2} \right) (0.5)(5 - 0.1 - 0.8)^2 = 147 \mu\text{A}$$

**Comment:** In the NMOS inverter with enhancement load, producing a relatively low output voltage  $V_{OL}$  requires a large difference in the sizes of the driver and load transistors. The load transistor size cannot be substantially reduced, so the power consumption also cannot be substantially reduced from the  $735 \mu\text{W}$  value. If an IC contained a modest 100,000 inverters and all inverters were conducting, the total required current to the IC would be 14.7 A and the total power dissipated would be 73.5 W!

### Example

The enhancement-load NMOS inverter shown in Fig. is biased at  $V_{DD} = 3$  V. The transistor parameters are  $V_{TND} = V_{TNL} = 0.4$  V,  $k'_n = 60 \text{ mA/V}^2$ ,  $(W/L)_D = 16$  and  $(W/L)_L = 2$ . (a) Find  $v_O$  when (i)  $v_I = 0$ , (ii)  $v_I = 2.6$ , (b) Calculate the power dissipated in the inverter when  $v_I = 2.6$  V.



### EX16.3

(a)

(i)

$$v_O = V_{DD} - V_{TNL} = 3 - 0.4$$

$$v_O = 2.6 \text{ V}$$

(ii)

$$\left( \frac{W}{L} \right)_D [2(v_I - 0.4)v_O - v_O^2] = \left( \frac{W}{L} \right)_L [V_{DD} - v_O - 0.4]^2$$

$$16[2(2.6 - 0.4)v_O - v_O^2] = 2[3 - v_O - 0.4]^2$$

$$35.2v_O - 8v_O^2 = 6.76 - 5.2v_O + v_O^2$$

$$9v_O^2 - 40.4v_O + 6.76 = 0$$

$$v_O = \frac{40.4 \pm \sqrt{1632.16 - 4(9)(6.76)}}{2(9)}$$

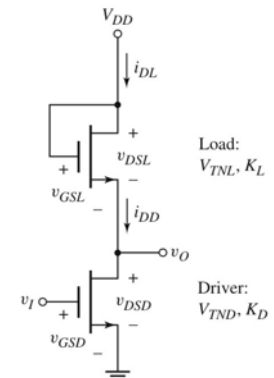
$$v_O = 0.174 \text{ V}$$

(b)

$$i_D = \left( \frac{60}{2} \right) (2)[3 - 0.174 - 0.4]^2$$

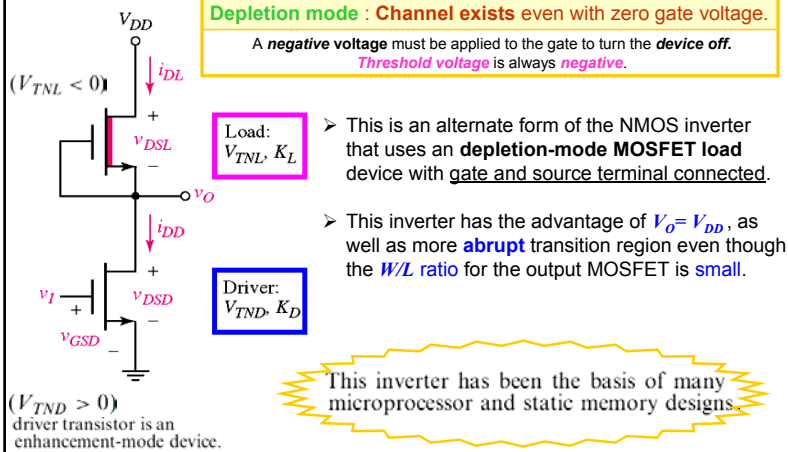
$$i_D = 353.1 \mu\text{A}$$

$$P = i_D \cdot V_{DD} = 1.06 \text{ mW}$$





## NMOS Inverter with Depletion Load



## N-Channel Depletion-Mode MOSFET

- In n-channel depletion mode MOSFET, an n-channel region or inversion layer exists under the gate oxide layer even at zero gate voltage and hence term **depletion mode**.

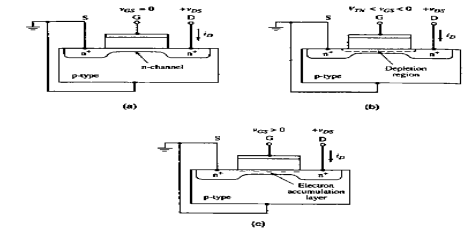
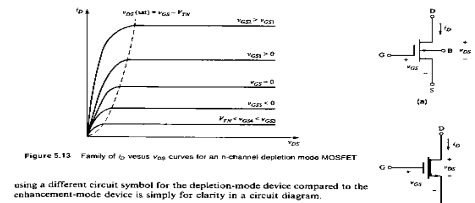
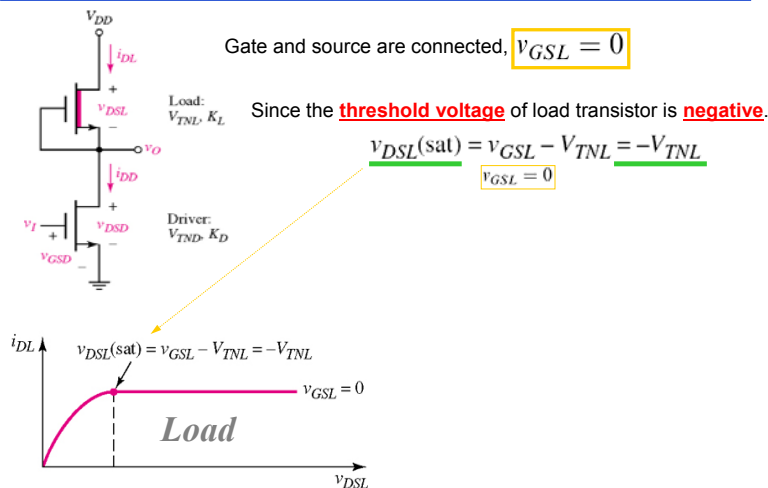


Figure 5.12 Cross section of an n-channel depletion-mode MOSFET for (a)  $V_{gs} = 0$ , (b)  $V_{gs} < 0$ , and (c)  $V_{gs} > 0$ .

- A **negative voltage** must be applied to the gate to turn the **device off**.
- The **threshold voltage** is always **negative** for this kind of device.

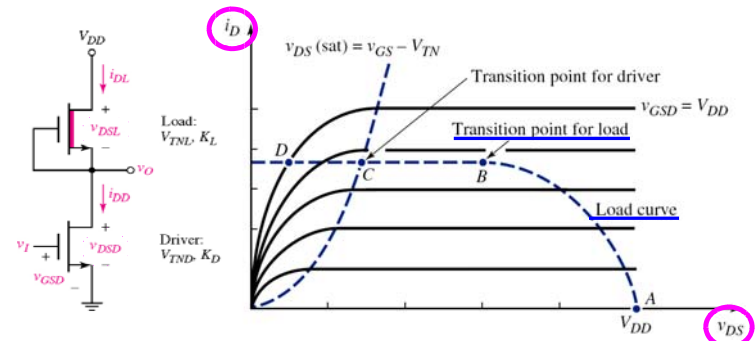


## NMOS Inverter with Depletion Load



## NMOS Inverter with Depletion Load

driver transistor characteristics and load curve



## NMOS Inverter with Depletion Load (cont.)

**Case I:** when  $V_I < V_{TND}$  (drive is cutoff): No drain current conduct in either transistor. That means the load transistor must be in the linear region of the operation and the output current can be expressed as follows

$$i_{DL}(\text{linear}) = K_L [2(V_{GSL} - V_{TNL})V_{DSL} - V_{DSL}^2]$$

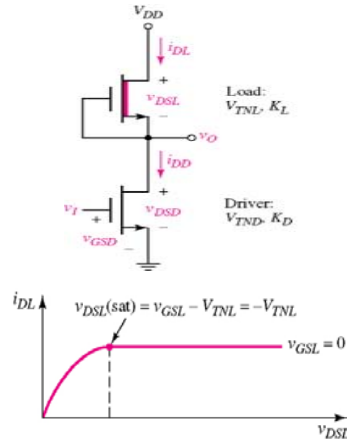
Since  $V_{GSL} = 0$ , and  $i_{DL} = 0$

$$0 = -K_L [2V_{TNL}V_{DSL} + V_{DSL}^2]$$

Which gives  $V_{DSL} = 0$  thus

$$V_O = V_{DD}$$

*This is the advantage of the depletion load inverter over the enhancement load inverter.*



## NMOS Inverter with Depletion Load

When  $v_I > V_{TND}$

driver turns on and in the saturation region; however, load is in the nonsaturation region. Q-point lies between points A and B

two drain currents equal,  $i_{DD} = i_{DL}$

$$K_D [v_{GSD} - V_{TND}]^2 = K_L [2(v_{GSL} - V_{TNL})v_{DSL} - v_{DSL}^2]$$

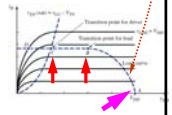
$$K_D [v_I - V_{TND}]^2 = K_L [2(-V_{TNL})(V_{DD} - v_O) - (V_{DD} - v_O)^2]$$

There are two transition points one for the load and one for the driver.

transition point for the load

$$v_{DSL} = V_{DD} - V_{OI} = v_{GSL} - V_{TNL} = -V_{TNL}$$

$$V_{OI} = V_{DD} + V_{TNL} \text{ Since } V_{TNL} \text{ is negative, the output voltage at the transition point is less than } V_{DD}.$$



## NMOS Inverter with Depletion Load

transition point for the driver

$$v_{DSD} = v_{GSD} - V_{TND}$$

$$v_{DS}(\text{sat}) = v_{GS} - V_{TN}$$

$$V_{OI} = V_{DD} - V_{TNL}$$

Load:  
 $V_{TNL}, K_L$

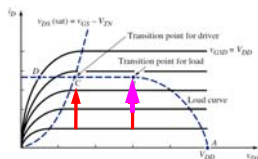
Driver:  
 $V_{TND}, K_D$

When the Q-point lies between points B and C both devices are in the saturation region,

$$K_D (v_{GSD} - V_{TND})^2 = K_L (v_{GSL} - V_{TNL})^2$$

$$\sqrt{\frac{K_D}{K_L}} (v_I - V_{TND}) = -V_{TNL}$$

This implies that input voltage is constant as the Q-point passes this region.



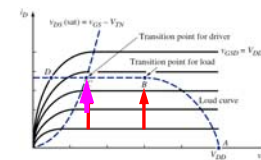
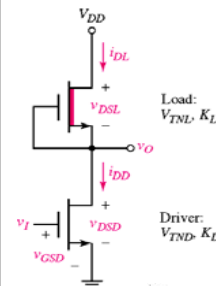
## NMOS Inverter with Depletion Load

driver is biased in the nonsaturation region while the load is in the saturation region. Q-point is between points C and D

$$K_D [2(v_{GSD} - V_{TND})v_{DSD} - v_{DSD}^2] = K_L (v_{GSL} - V_{TNL})^2$$

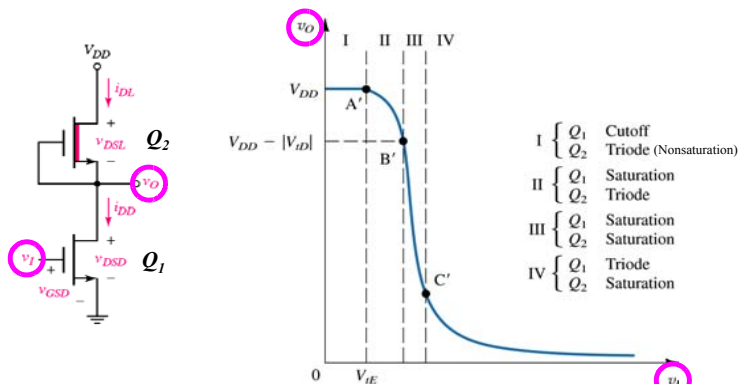
$$\frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

This implies that input and output voltages are not linear in this region.



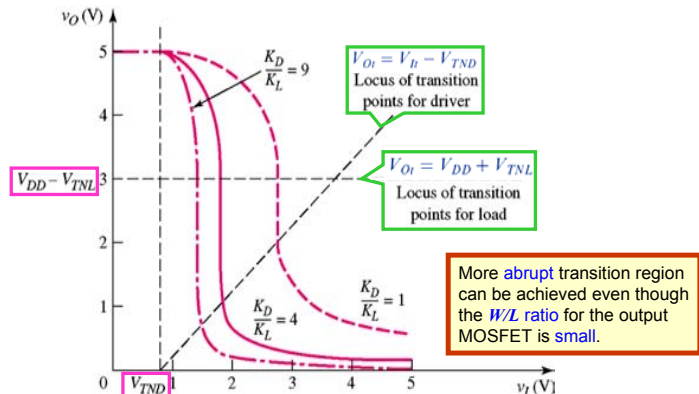
## NMOS Inverter with Depletion Load

Voltage transfer characteristics,  
NMOS inverter with depletion load,



## NMOS Inverter with Depletion Load

Voltage transfer characteristics,  
NMOS inverter with depletion load, for three aspect ratios



### Example 16.4

P1014

**Design Example 16.4 Objective:** Design the aspect ratio  $K_D/K_L$  to produce a specified low output voltage, and determine the power dissipation in the inverter with depletion load for a minimum  $W/L$  ratio for the load transistor.

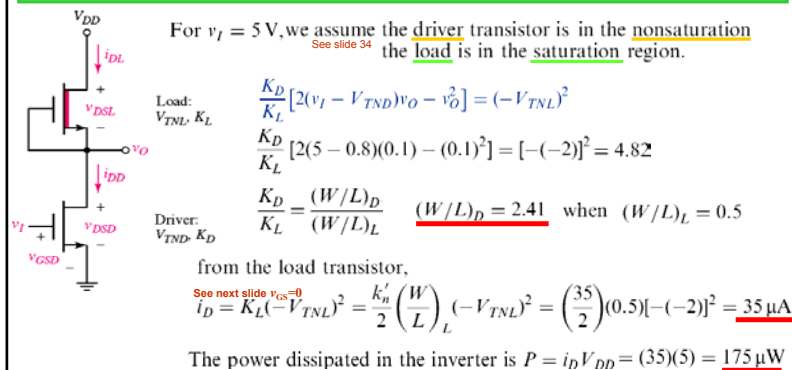
Consider the inverter in Figure 16.10(a) biased at  $V_{DD} = 5$  V. The transistor parameters are:  $V_{TND} = 0.8$  V,  $V_{TNL} = -2$  V, and  $k'_n = 35 \mu\text{A}/\text{V}^2$ . Determine  $K_D/K_L$  such that  $v_O = 0.10$  V when  $v_I = 5$  V. Determine  $(W/L)_D$  and the power dissipation in the inverter for  $(W/L)_L = 0.5$ .

### Example 16.4

P1014

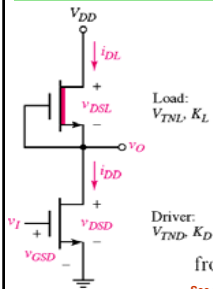
**Design Example 16.4 Objective:** Design the aspect ratio  $K_D/K_L$  to produce a specified low output voltage, and determine the power dissipation in the inverter with depletion load for a minimum  $W/L$  ratio for the load transistor.

Consider the inverter in Figure 16.10(a) biased at  $V_{DD} = 5$  V. The transistor parameters are:  $V_{TND} = 0.8$  V,  $V_{TNL} = -2$  V, and  $k'_n = 35 \mu\text{A}/\text{V}^2$ . Determine  $K_D/K_L$  such that  $v_O = 0.10$  V when  $v_I = 5$  V. Determine  $(W/L)_D$  and the power dissipation in the inverter for  $(W/L)_L = 0.5$ .



**Design Example 16.4 Objective:** Design the aspect ratio  $K_D/K_L$  to produce a specified low output voltage, and determine the power dissipation in the inverter with depletion load for a minimum  $W/L$  ratio for the load transistor.

Consider the inverter in Figure 16.10(a) biased at  $V_{DD} = 5\text{ V}$ . The transistor parameters are:  $V_{TND} = 0.8\text{ V}$ ,  $V_{TNL} = -2\text{ V}$ , and  $k'_n = 35\text{ }\mu\text{A/V}^2$ . Determine  $K_D/K_L$  such that  $v_O = 0.10\text{ V}$  when  $v_I = 5\text{ V}$ . Determine  $(W/L)_D$  and the power dissipation in the inverter for  $(W/L)_L = 0.5$ .



from the load transistor,

$$i_D = K_L(-V_{TNL})^2 = \frac{k'_n}{2} \left(\frac{W}{L}\right)_L (-V_{TNL})^2 = \left(\frac{35}{2}\right)(0.5)[-(-2)]^2 = \underline{35\text{ }\mu\text{A}}$$

The power dissipated in the inverter is  $P = i_D V_{DD} = (35)(5) = \underline{175\text{ }\mu\text{W}}$

## Summary of NMOS inverter with Resister Load

### Current-Voltage Relationship

$$\text{Saturation Region} \quad i_D = K_n(v_{GS} - V_{TN})^2 = K_n(v_I - V_{TN})^2$$

$$v_O = V_{DD} - K_n R_D (v_I - V_{TN})^2$$

$$\text{Transition Region} \quad V_{OI} = V_h - V_{TN}$$

$$K_n R_D (V_h - V_{TN})^2 + (V_h - V_{TN}) - V_{DD} = 0$$

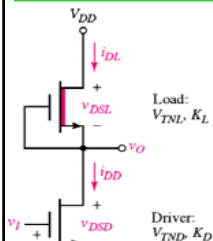
$$\text{Nonsaturation Region}$$

$$i_D = K_n[2(v_I - v_{TN})v_O - v_O^2]$$

$$v_O = V_{DD} - K_n R_D[2(v_I - V_{TN})v_O - v_O^2]$$

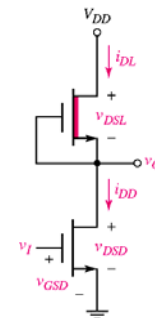
**Design Example 16.4 Objective:** Design the aspect ratio  $K_D/K_L$  to produce a specified low output voltage, and determine the power dissipation in the inverter with depletion load for a minimum  $W/L$  ratio for the load transistor.

Consider the inverter in Figure 16.10(a) biased at  $V_{DD} = 5\text{ V}$ . The transistor parameters are:  $V_{TND} = 0.8\text{ V}$ ,  $V_{TNL} = -2\text{ V}$ , and  $k'_n = 35\text{ }\mu\text{A/V}^2$ . Determine  $K_D/K_L$  such that  $v_O = 0.10\text{ V}$  when  $v_I = 5\text{ V}$ . Determine  $(W/L)_D$  and the power dissipation in the inverter for  $(W/L)_L = 0.5$ .

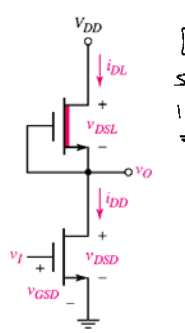


**Comment:** A relatively low output voltage  $V_{OL}$  can be produced in the NMOS inverter with depletion load, even when the load and driver transistors are not vastly different in size. The power dissipation in this inverter is also substantially less than in the enhancement-load inverter since the aspect ratio is smaller.

**D16.5** Consider the depletion load inverter in Figure 16.10(a) biased at  $V_{DD} = 5\text{ V}$ . The threshold voltages are  $V_{TND} = 0.8\text{ V}$  and  $V_{TNL} = -2\text{ V}$ . Design the inverter such that the maximum power dissipation is  $350\text{ }\mu\text{W}$  and the output voltage is  $0.05\text{ V}$  when  $v_I = 5\text{ V}$ . (Ans.  $(W/L)_L = 1$ ,  $(W/L)_D = 9.58$ )



**D16.5** Consider the depletion load inverter in Figure 16.10(a) biased at  $V_{DD} = 5$  V. The threshold voltages are  $V_{TND} = 0.8$  V and  $V_{TNL} = -2$  V. Design the inverter such that the maximum power dissipation is  $350 \mu\text{W}$  and the output voltage is  $0.05$  V when  $v_I = 5$  V. (Ans.  $(W/L)_L = 1$ ,  $(W/L)_D = 9.58$ )



$P = i_D V_{DD}$   
 $350 = i_D \cdot 5 \Rightarrow i_D = \frac{350}{5} = 70 \mu\text{A}$   
 $i_D = 70 \mu\text{A}$  (Load)

Since gate to source terminal of the load transistors are short  
 $\Rightarrow V_{GS_L} = 0$ , which is much greater than  $V_{TNL}$

Load transistor is in Saturation mode

$$i_D = K_L (V_{GS_L} - V_{TNL})^2$$

Since  $V_{GS_L} = 0$

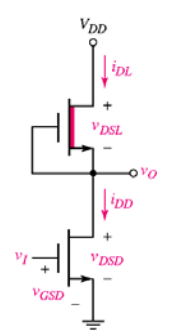
$$i_D = K_L (0 - V_{TNL})^2 = K_L (-V_{TNL})^2$$

$$i_D = \left(\frac{K_n}{2}\right) \left(\frac{W}{L}\right)_L (-V_{TNL})^2$$

$$70 = \left(\frac{35}{2}\right) \left(\frac{W}{L}\right)_L (-(-2))^2$$

$$\left(\frac{W}{L}\right)_L = 1$$

**D16.5** Consider the depletion load inverter in Figure 16.10(a) biased at  $V_{DD} = 5$  V. The threshold voltages are  $V_{TND} = 0.8$  V and  $V_{TNL} = -2$  V. Design the inverter such that the maximum power dissipation is  $350 \mu\text{W}$  and the output voltage is  $0.05$  V when  $v_I = 5$  V. (Ans.  $(W/L)_L = 1$ ,  $(W/L)_D = 9.58$ )



Also we have  
 $V_I = 5$  V and  $V_O = 0.05$  V for Driver  
 $\Rightarrow$  Driver must be in non-saturation mode

$$i_D = K_D [(2(V_{GS_D} - V_{TND})V_{DS_D} - V_{DS_D}^2)]$$

or

$$i_D = K_D [2(V_I - V_{TND})V_O - V_O^2]$$

Substituting given values

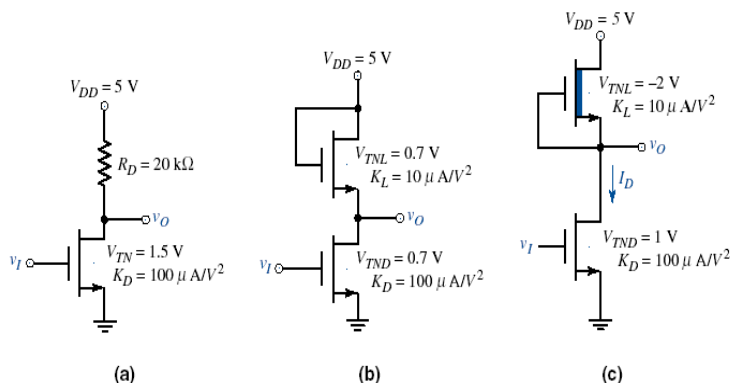
$$70 = \frac{K_n}{2} \left(\frac{W}{L}\right)_D [ \dots ]$$

$$70 = \frac{35}{2} \left(\frac{W}{L}\right)_D [2(5 - 0.8)0.05 - (0.05)^2]$$

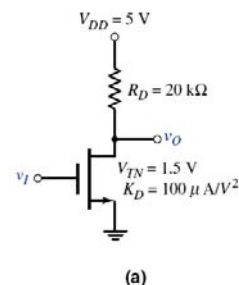
$$70 = 7.31 \left(\frac{W}{L}\right)_D$$

$$\Rightarrow \left(\frac{W}{L}\right)_D = 9.58$$

**16.14** Calculate the power dissipated in each inverter circuit in Figure P16.14 for the following input conditions: (a) Inverter a: (i)  $v_I = 0.5$  V, (ii)  $v_I = 5$  V; (b) Inverter b: (i)  $v_I = 0.25$  V, (ii)  $v_I = 4.3$  V; (c) Inverter c: (i)  $v_I = 0.03$  V, (ii)  $v_I = 5$  V.



**16.14** Calculate the power dissipated in each inverter circuit in Figure P16.14 for the following input conditions: (a) Inverter a: (i)  $v_I = 0.5$  V, (ii)  $v_I = 5$  V; (b) Inverter b: (i)  $v_I = 0.25$  V, (ii)  $v_I = 4.3$  V; (c) Inverter c: (i)  $v_I = 0.03$  V, (ii)  $v_I = 5$  V.



(i)  $v_I = 0.5$  V  $\Rightarrow i_D = 0 \Rightarrow P = 0$

(ii)  $v_I = 5$  V, From Equation (16.12),  
non-saturation region

$$v_O = V_{DD} - K_n R_D [2(v_I - V_{TN})v_O - v_O^2]$$

$$v_O = 5 - (0.1)(20)[2(5 - 1.5)v_O - v_O^2]$$

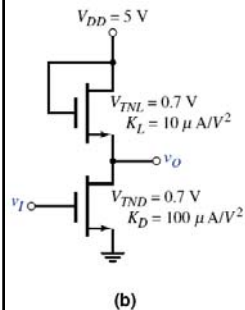
$$2v_O^2 - 15v_O + 5 = 0$$

$$v_O = \frac{15 \pm \sqrt{(15)^2 - 4(2)(5)}}{2(2)} \Rightarrow v_O = 0.35 \text{ V}$$

$$i_D = \frac{5 - 0.35}{20} = 0.2325 \text{ mA}$$

$$P = i_D \cdot V_{DD} = (0.2325)(5) \Rightarrow P = 1.16 \text{ mW}$$

**16.14** Calculate the power dissipated in each inverter circuit in Figure P16.14 for the following input conditions: (a) Inverter a: (i)  $v_I = 0.5$  V, (ii)  $v_I = 5$  V; (b) Inverter b: (i)  $v_I = 0.25$  V, (ii)  $v_I = 4.3$  V; (c) Inverter c: (i)  $v_I = 0.03$  V, (ii)  $v_I = 5$  V.



(i)  $v_I = 0.25$  V  $\Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

(ii)  $v_I = 4.3$  V, From Equation (16.23),

input voltage greater than  $V_{th}$

$$K_D[2(v_I - V_{TND})v_O - v_O^2] = K_L(V_{DD} - v_O - V_{TNL})^2$$

$$100[2(4.3 - 0.7)v_O - v_O^2] = 10[5 - v_O - 0.7]^2$$

$$10[7.2v_O - v_O^2] = 18.49 - 8.6v_O + v_O^2$$

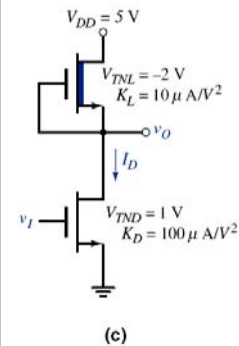
$$11v_O^2 - 80.6v_O + 18.49 = 0$$

$$v_O = \frac{80.6 \pm \sqrt{(80.6)^2 - 4(11)(18.49)}}{2(11)} = 0.237 \text{ V}$$

$$i_D = 10[5 - 0.237 - 0.7]^2 = 165 \text{ } \mu\text{A}$$

$$P = i_D \cdot V_{DD} = (165)(5) \Rightarrow \underline{P = 825 \text{ } \mu\text{W}}$$

**16.14** Calculate the power dissipated in each inverter circuit in Figure P16.14 for the following input conditions: (a) Inverter a: (i)  $v_I = 0.5$  V, (ii)  $v_I = 5$  V; (b) Inverter b: (i)  $v_I = 0.25$  V, (ii)  $v_I = 4.3$  V; (c) Inverter c: (i)  $v_I = 0.03$  V, (ii)  $v_I = 5$  V.



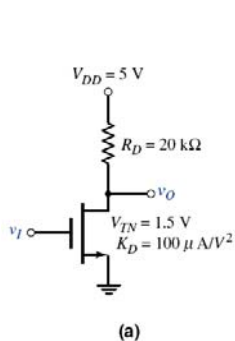
(i)  $v_I = 0.03$  V  $\Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

(ii)  $v_I = 5$  V

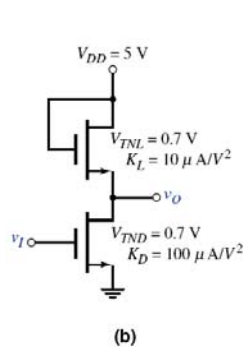
$$i_D = K_L(-V_{TNL})^2 = (10)[-(-2)]^2 = 40 \text{ } \mu\text{A}$$

$$P = i_D \cdot V_{DD} = (40)(5) \Rightarrow \underline{P = 200 \text{ } \mu\text{W}}$$

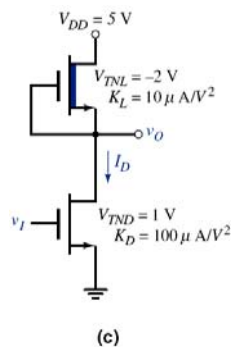
### NMOS Inverter



**1160 μW**  
Resistor Load



**825 μW**  
Enhancement Load



**200 μW**  
Depletion Load

## Chapter 16

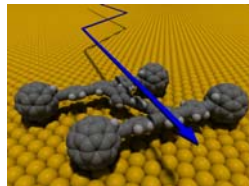
# MOSFET Digital Circuits

### Chapter 16.2

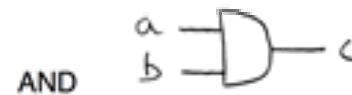
## NMOS Logic Circuit

## NMOS Logic Circuit

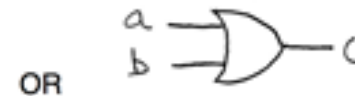
NMOS logic circuits are constructed by connecting driver transistor in **parallel, series** or **series-parallel combinations** to produce required output logic function



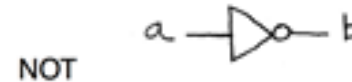
## Logic Gates



ab	c
00	0
01	0
10	0
11	1

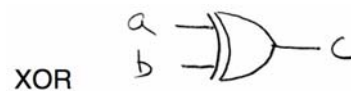


ab	c
00	0
01	1
10	1
11	1

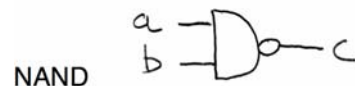


a	b
0	1
1	0

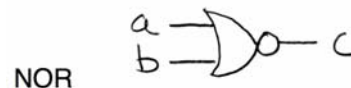
## Logic Gates



ab	c
00	0
01	1
10	1
11	0



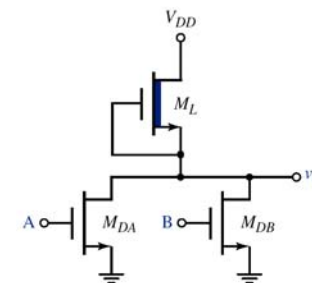
ab	c
00	1
01	1
10	1
11	0



ab	c
00	1
01	0
10	0
11	0

## NMOS NOR Gate

- NMOS NOR gate can be constructed by connecting an additional **driver transistor in parallel** with a **depletion load inverter**.



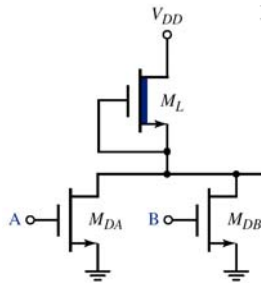
Two-input NMOS NOR logic gate with depletion load

If  $A = B = \text{logic } 0$ ,  
then both  $M_{DA}$  and  $M_{DB}$  are cut off  
and  $v_O = V_{DD}$ .

If  $A = \text{logic } 1$  and  $B = \text{logic } 0$ ,  
then  $M_{DB}$  is cut off  
and the NMOS inverter with  $M_L$  and  $M_{DA}$   
if  $A = \text{logic } 0$  and  $B = \text{logic } 1$ ,  
same inverter configuration.



## NMOS NOR Gate



If  $A = B = \text{logic } 1$ ,  
then both  $M_{DA}$  and  $M_{DB}$  turn on  
and the two driver transistors are  
effectively in parallel.

The value of the output voltage changes slightly.

$$i_{DL} = i_{DA} + i_{DB}$$

$$K_L[v_{GSL} - V_{TNL}]^2 = K_{DA}[2(v_{GSA} - V_{TNA})v_{DSA} - v_{DSA}^2] \\ + K_{DB}[2(v_{GSB} - V_{TNB})v_{DSB} - v_{DSB}^2]$$

If transistors are identical,

$$K_{DA} = K_{DB} \equiv K_D \quad V_{TNA} = V_{TNB} \equiv V_{TND}$$

Two-input NMOS NOR logic gate  
with depletion load

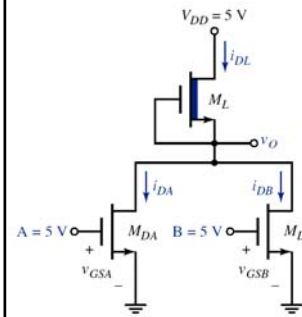
$$v_{GSL} = 0, v_{GSA} = v_{GSB} = V_{DD}, v_{DSA} = v_{DSB} = v_O$$

$$[-V_{TNL}]^2 = 2\left(\frac{K_D}{K_L}\right)[2(V_{DD} - V_{TND})v_O - v_O^2]$$

when both drivers are conducting, the effective width-to-length ratio of the composite driver transistor doubles.  
the output voltage becomes slightly smaller when both inputs are high.

## NMOS NOR Gate

When all Inputs are at logic 1



When  $A = B = \text{logic } 1$

Both driver transistors are switched into nonsaturation region and load transistor is biased in saturation region.

$$i_{DL} = i_{DA} + i_{DB}$$

$$K_L[v_{GSL} - V_{TNL}]^2 = K_{DA}[2(v_{GSA} - V_{TNA})v_{DSA} - v_{DSA}^2] \\ + K_{DB}[2(v_{GSB} - V_{TNB})v_{DSB} - v_{DSB}^2]$$

Suppose two driver transistors are identical,  
 $K_{DA} = K_{DB} \equiv K_D$   $V_{TNA} = V_{TNB} = V_{TND}$   
 $v_{GSL} = 0, v_{GSA} = v_{GSB} = V_{DD}, v_{DSA} = v_{DSB} = v_O$

$$[-V_{TNL}]^2 = 2\left(\frac{K_D}{K_L}\right)[2(V_{DD} - V_{TND})v_O - v_O^2]$$

when both drivers are conducting,  
the effective width-to-length ratio of  
the composite driver transistor doubles.

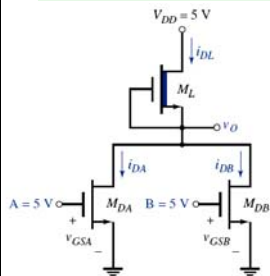
This means that the output voltage  
becomes slightly smaller when both inputs are high.  
*higher the aspect ratio lower the output.*

Example 16.7

p1030

**Example 16.7** Objective: Determine the low output voltage of an NMOS NOR circuit.

Consider the NOR circuit in Figure 16.24 biased at  $V_{DD} = 5 \text{ V}$ . Assume that  $k_n' = 35 \mu\text{A}/\text{V}^2$ . Also assume the width-to-length ratios of the load and driver transistors are  $(W/L)_L = 1$  and  $(W/L)_D = 4$ , respectively. Let  $V_{TND} = 0.8 \text{ V}$  and  $V_{TNL} = -2 \text{ V}$ . Neglect the body effect.



Two-input NMOS NOR logic gate

If,  $A = \text{logic } 1 = 5 \text{ V}$  and  $B = \text{logic } 0$ , then  $M_{DB}$  is cut off.

$$\frac{K_D}{K_L}[2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

$$\frac{K_D}{K_L}[2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2 \quad \left(\frac{4}{1}\right)[2(5 - 0.8)v_O - v_O^2] = (2)^2$$

$$v_O = 0.121 \text{ V}$$

If both inputs go high,

$$A = B = V_{DD} = 5 \text{ V}$$

$$[-V_{TNL}]^2 = 2\left(\frac{K_D}{K_L}\right)[2(V_{DD} - V_{TND})v_O - v_O^2]$$

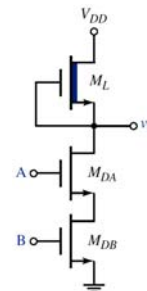
$$[-V_{TNL}]^2 = 2\left(\frac{K_D}{K_L}\right)[2(V_{DD} - V_{TND})v_O - v_O^2]$$

$$(2)^2 = 2\left(\frac{4}{1}\right)[2(5 - 0.8)v_O - v_O^2]$$

$$v_O = 0.060 \text{ V}$$

## NMOS NAND Gate

Additional driver transistor connected in Series



Two-input NMOS NAND logic gate  
with depletion load

If both  $A = B = \text{logic } 0$ , or if either  $A$  or  $B$  is a logic 0,  
at least one driver is cut off, and the output is high.

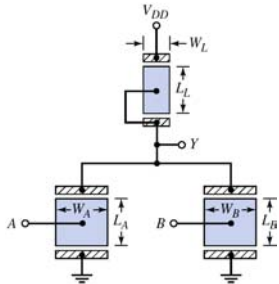
If both  $A = B = \text{logic } 1$ ,  
then the composite driver of the NMOS inverter conducts  
and the output goes low.

Since the gate-to-source voltages of  $M_{DA}$  and  $M_{DB}$  are not equal, determining the actual voltage  $V_{OL}$  of a NAND gate is difficult. The drain-to-source voltages of  $M_{DA}$  and  $M_{DB}$  must adjust themselves to produce the same current.

## NMOS Logic Circuit

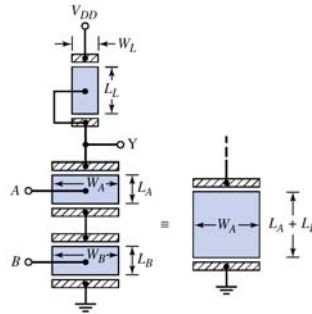
### Concept of Effective Width-to-Length Ratios

#### Parallel combination



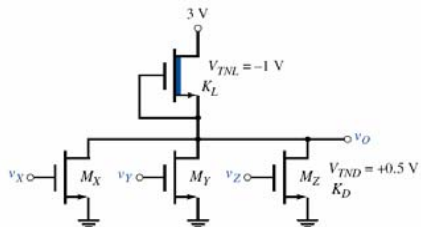
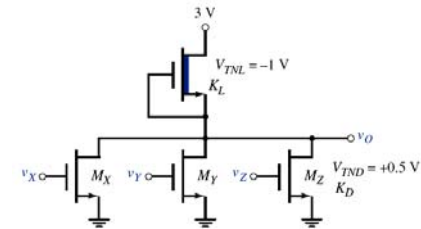
For the **NOR gate** the **effective width of the drivers transistors doubles**.  
The effective aspect ratio is increased.

#### Series combination



For the **NAND gate** the **effective length of the driver transistors doubles**.  
The effective aspect ratio is decreased.

**D16.20** Consider the three-input NOR logic gate in Figure P16.20. The transistor parameters are  $V_{TNL} = -1 \text{ V}$  and  $V_{TND} = 0.5 \text{ V}$ . The maximum value of  $v_O$  in its low state is to be  $0.1 \text{ V}$ . (a) Determine  $K_D/K_L$ . (b) The maximum power dissipation in the NOR logic gate is to be  $0.1 \text{ mW}$ . Determine the width-to-length ratios of the transistors. (c) Determine  $v_O$  when  $v_X = v_Y = v_Z = 3 \text{ V}$ .



- (a) Maximum value of  $v_O$  in low state- when only one input is high, then,

$$\frac{K_D}{K_L} [2(v_I - V_{TND})v_O - v_O^2] = (-V_{TNL})^2$$

$$\frac{K_D}{K_L} [2(3 - 0.5)(0.1) - (0.1)^2] = [ -(-1) ]^2 = 2.04$$

- (b)  $P = i_D \cdot V_{DD}$

$$0.1 = i_D(3) \Rightarrow i_D = 33.3 \mu\text{A}$$

$$i_D = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right)_L (-V_{TNL})^2$$

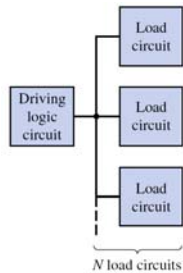
$$33.3 = \left( \frac{80}{2} \right) \left( \frac{W}{L} \right)_L [ -(-1) ]^2 \Rightarrow \left( \frac{W}{L} \right)_L = 0.8325$$

$$\left( \frac{W}{L} \right)_D = 1.70$$

- (c)  $[-V_{TNL}]^2 = 2 \left( \frac{K_D}{K_L} \right) [2(V_{DD} - V_{TND})v_O - v_O^2]$

$$3(2.04) [2(3 - 0.5)v_O - v_O^2] = [ -(-1) ]^2 \Rightarrow v_O = 0.0329 \text{ V}$$

## Fan-In and Fan-Out



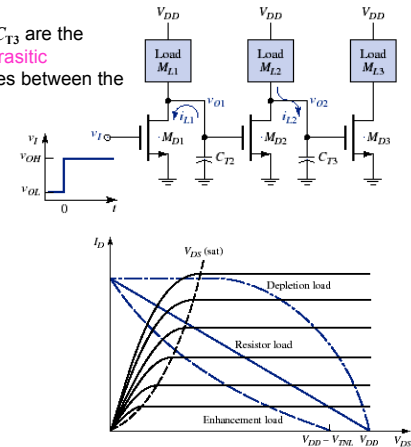
Logic circuit driving  $N$  load circuits

- The **Fan-in** of a gate is the number of its inputs. Thus a four input NOR gate has a fan-in of 4.
- Similarly, **Fan-Out** is the maximum number of similar gates that a gate can drive while remaining within guaranteed specifications.

dc characteristics of MOS logic circuits are unaffected by the fanout to other MOS logic inputs. However, the load capacitance due to a large fanout seriously degrades the switching speed and propagation delay times. Consequently, maintaining the propagation delay time below a specified maximum value determines the fanout of MOS digital circuits.

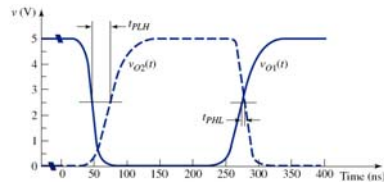
## Transient Analysis of NMOS Inverters

- The source of capacitance  $C_{T2}$  and  $C_{T3}$  are the transistor **input capacitances** and **parasitic capacitances** due to interconnect lines between the inverter stages.
- The constant current over a wide range of  $V_{DS}$  provided by the depletion load implies that this type of inverter switch a capacitive load more rapidly than the other two types inverter configurations.

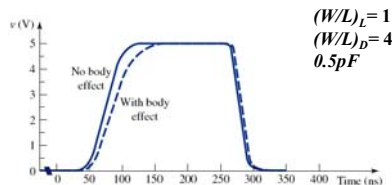


The rate at which the voltage across a load capacitance changes is a direct function of the current through the capacitance.

## Transient Analysis of NMOS Inverters



- The **raise time** is longer because the load capacitor is charged by the current through the smaller load transistor.



- The **fall time** relatively short, because the load capacitor discharges through the large driver transistor.

Switching characteristics of an NMOS inverter with depletion load

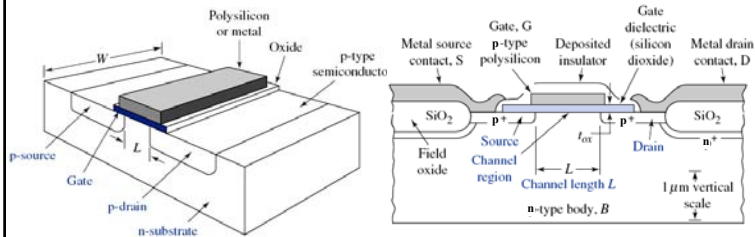
## Chapter 16

## MOSFET Digital Circuits

### Chapter 16.3

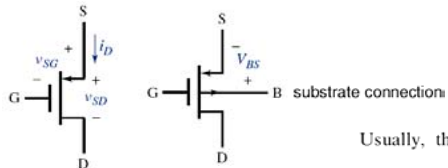
### CMOS Inverter

## p-Channel MOSFET



the channel length is the same for all transistors, while the channel width is variable.

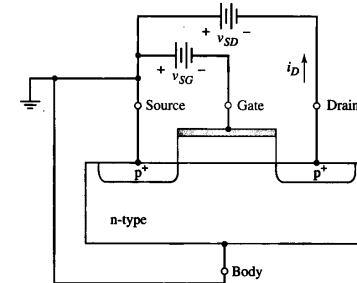
### p-channel enhancement-mode MOSFET



Usually, the p-channel depletion-mode device is not used in CMOS digital circuits

## p-Channel MOSFET

- In p-channel enhancement device. A negative gate-to-source voltage must be applied to create the inversion layer, or **channel region**, of holes that, "connect" the source and drain regions.
- The **threshold voltage**  $V_{TP}$  for **p-channel enhancement-mode** device is always **negative** and **positive** for **depletion-mode** PMOS.



Cross-section of p-channel enhancement mode MOSFET

## Summary of Transistor Operation

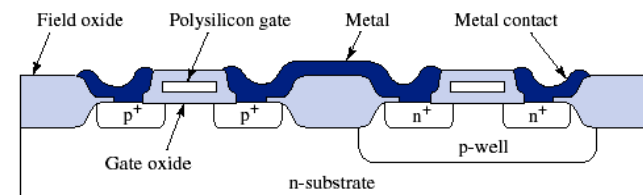
NMOS	PMOS
Nonsaturation region ( $v_{DS} < v_{DS}(\text{sat})$ )	Nonsaturation region ( $v_{SD} < v_{SD}(\text{sat})$ )
$i_D = K_n[2(v_{GS} - V_{TN})v_{DS} - v_{DS}^2]$	$i_D = K_p[2(v_{SG} + V_{TP})v_{SD} - v_{SD}^2]$
Saturation region ( $v_{DS} > v_{DS}(\text{sat})$ )	Saturation region ( $v_{SD} > v_{SD}(\text{sat})$ )
$i_D = K_n(v_{GS} - V_{TN})^2$	$i_D = K_p(v_{SG} + V_{TP})^2$
Transition point	Transition point
$v_{DS}(\text{sat}) = v_{GS} - V_{TN}$	$v_{SD}(\text{sat}) = v_{SG} + V_{TP}$
Enhancement mode	Enhancement mode
$V_{TN} > 0$	$V_{TP} < 0$
Depletion mode	Depletion mode
$V_{TN} < 0$	$V_{TP} > 0$

## CMOS

### Complementary MOS

The most abundant devices on earth

- Although the processing is more complicated for CMOS circuits than for NMOS circuits, CMOS has replaced NMOS at all level of integration, in both analog and digital applications.
- The basic reason of this replacement is that the **power dissipation in CMOS logic circuits** is much less than in NMOS circuits.



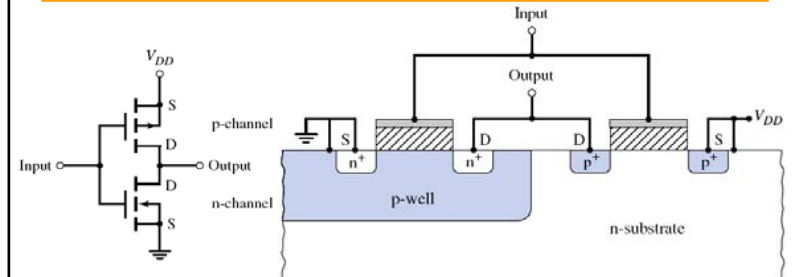
## CMOS Properties

- ❑ Full rail-to-rail swing → high noise margins
  - Logic levels not dependent upon the relative device sizes → transistors can be minimum size → ratio less
- ❑ Always a path to  $V_{DD}$  or GND in steady state → low output impedance (output resistance in  $k\Omega$  range) → large fan-out.
- ❑ Extremely high input resistance (gate of MOS transistor is near perfect insulator) → nearly zero steady-state input current
- ❑ No direct path steady-state between power and ground → no static power dissipation
- ❑ Propagation delay function of load capacitance and resistance of transistors

## CMOS Inverter

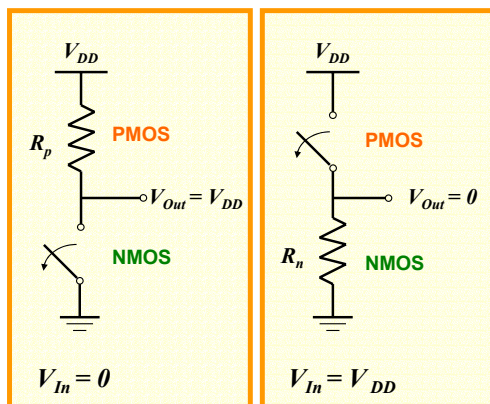
- In the fabrication process, a separate **p-well** region is formed within the starting n-substrate.
- The n-channel MOSFET is fabricated in the p-well region and p-channel MOSFET is fabricated in the n-substrate.

a series combination of a p-channel and an n-channel MOSFET.



## CMOS Inverter

### Steady State Response

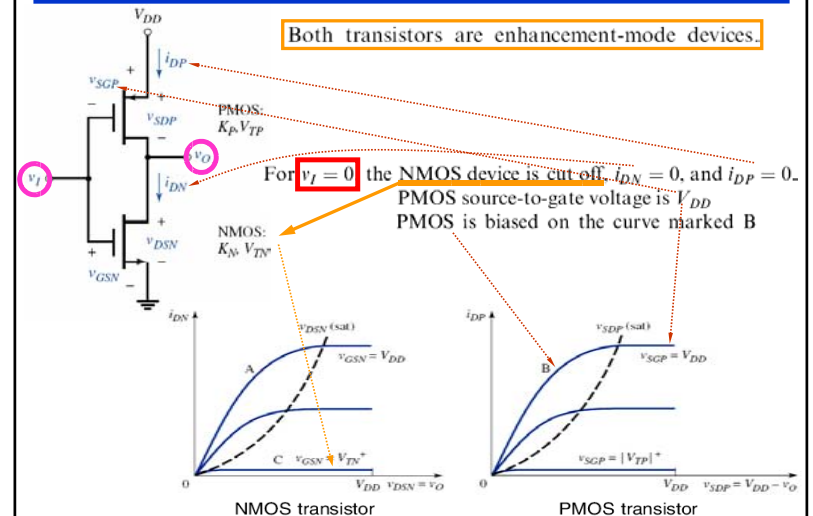


$$V_{OL} = 0$$

$$V_{OH} = V_{DD}$$

## DC Analysis of the CMOS Inverter

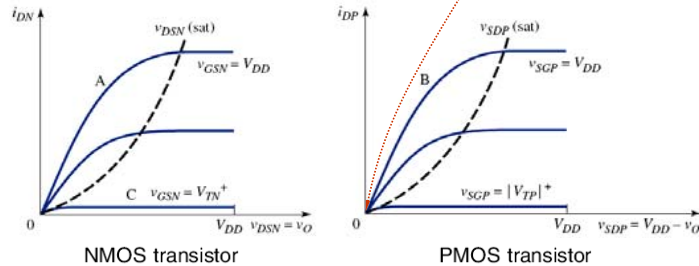
Both transistors are enhancement-mode devices.



## DC Analysis of the CMOS Inverter

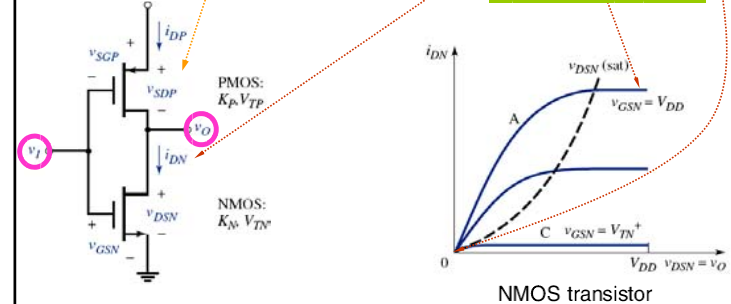
For  $v_I = 0$  the NMOS device is cut off,  $i_{DN} = 0$ , and  $i_{DP} = 0$ .  
 PMOS source-to-gate voltage is  $V_{DD}$   
 PMOS is biased on the curve marked B

Since the only point on the curve corresponding to  $i_{DP} = 0$  occurs at  $v_{SDP} = 0 = V_{DD} - v_O$ , the output voltage is  $v_O = V_{DD}$ . This condition exists as long as the NMOS transistor is cut off, or  $v_I \leq V_{TN}$ .

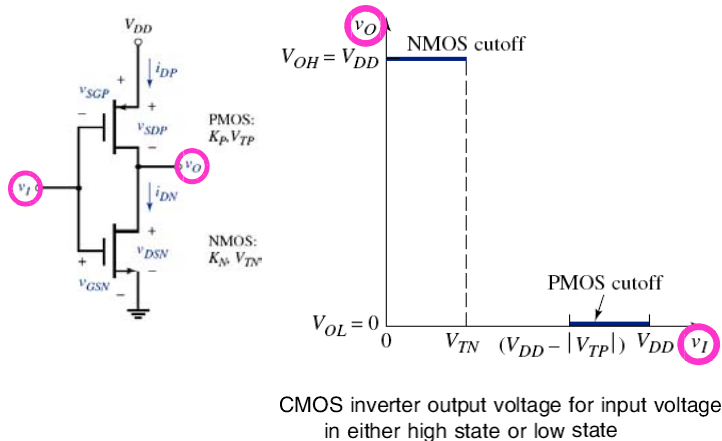


## DC Analysis of the CMOS Inverter

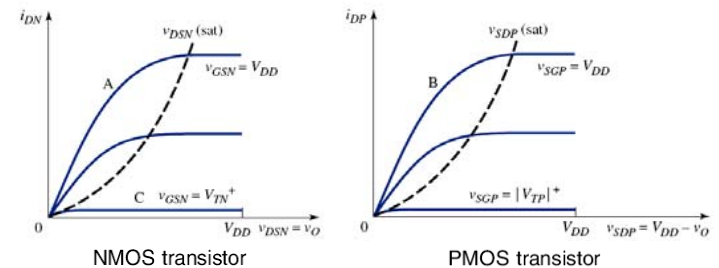
For  $v_I = V_{DD}$  the PMOS device is cut off,  $i_{DP} = 0$ , and  $i_{DN} = 0$ .  
 NMOS gate-to-source voltage is  $V_{DD}$   
 NMOS is biased on the curve marked A  
 The only point on the curve corresponding to  $i_{DN} = 0$  occurs at  $v_{DSN} = v_O = 0$ .  
 The output voltage is zero as long as the PMOS transistor is cut off, or  $v_{SGP} = V_{DD} - v_I \leq |V_{TP}|$ .  
 input voltage is in the range  $V_{DD} - |V_{TP}| \leq v_I \leq V_{DD}$ .



## DC Analysis of the CMOS Inverter



## DC Analysis of the CMOS Inverter



Voltage Transfer Curve

**EX16.2**

(a)

$$v_o = V_{DD} - I_D R_D$$

$$v_o = 3 - \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) [2(3 - 0.5)v_o - v_o^2] R_D$$

$$v_o = 0.1$$

$$0.1 = 3 - \left( \frac{0.06}{2} \right) (5) [(5)(0.1) - (0.1)^2] R_D$$

$$0.1 = 3 - 0.0735 R_D$$

$$R_D = 39.5 \text{ K}$$

(b)

$$\left( \frac{0.06}{2} \right) (5) (39.5) (V_B - 0.5)^2 + (V_B - 0.5) - 3 = 0$$

$$5.925 (V_B - 0.5)^2 + (V_B - 0.5) - 3 = 0$$

$$(V_B - 0.5) = V_{Ot} = \frac{-1 \pm \sqrt{1 + 4(5.925)(3)}}{2(5.925)}$$

$$V_{Ot} = 0.632 \text{ V}$$

$$V_B = 1.132 \text{ V}$$

16.6

(a) From Equation (16.23)

$$\frac{K_D}{K_L} [2(3 - 0.5)(0.25) - (0.25)^2] = (3 - 0.25 - 0.5)^2 \Rightarrow \frac{K_D}{K_L} = 4.26$$

$$(b) \quad \frac{K_D}{K_L} [2(2.5 - 0.5)(0.25) - (0.25)^2] = (3 - 0.25 - 0.5)^2 \Rightarrow \frac{K_D}{K_L} = 5.4$$

$$(c) \quad i_D = K_L (V_{GSL} - V_{TNL})^2 = K_L (V_{DD} - v_o - V_{TNL})^2$$

$$= \left( \frac{0.080}{2} \right) (1)(3 - 0.25 - 0.5)^2 \Rightarrow i_D = 0.203 \text{ mA}$$

$$P = i_D \cdot V_{DD} = (0.203)(3) \Rightarrow P = 0.608 \text{ mW}$$

for both parts (a) and (b).

**HW solution****16.4**

$$0.25 = I(33) \Rightarrow I = 75.76 \text{ } \mu\text{A}$$

$$R = \frac{3.3 - 0.15}{0.07576} \Rightarrow R = 41.6 \text{ K}$$

$$I = \left( \frac{k'_n}{2} \right) \left( \frac{W}{L} \right) (V_{GS} - V_{TN})^2$$

$$75.76 = \left( \frac{80}{2} \right) \left( \frac{W}{L} \right) (3.3 - 0.8)^2 \Rightarrow \left( \frac{W}{L} \right) = 0.303$$

(b)

$$V_{DS}(\text{sat}) = V_{GS} - V_{TN}$$

$$I_D = K_n (V_{GS} - V_{TN})^2 = \frac{V_{DD} - V_{DS}(\text{sat})}{R}$$

$$\left( \frac{0.08}{2} \right) (0.303) (V_{GS}^2 - 1.6V_{GS} + 0.64) = \frac{3.3 - (V_{GS} - 0.8)}{41.6}$$

$$0.504(V_{GS}^2 - 1.6V_{GS} + 0.64) = 4.1 - V_{GS}$$

$$0.504V_{GS}^2 + 0.1936V_{GS} - 3.777 = 0$$

$$V_{GS} = \frac{-0.1936 \pm \sqrt{0.03748 + 4(0.504)(3.777)}}{2(0.504)}$$

$$V_{GS} = 2.55 \text{ V}$$

$$\text{For } 0.8 \leq V_{GS} \leq 2.55 \text{ V}$$

Transistor biased in saturation region

16.9

$$V_{OH} = V_B - V_{TN} = \text{Logic 1}$$

So

$$(a) \quad V_B = 4 \text{ V} \Rightarrow V_{OH} = 3 \text{ V}$$

$$(b) \quad V_B = 5 \text{ V} \Rightarrow V_{OH} = 4 \text{ V}$$

$$(c) \quad V_B = 6 \text{ V} \Rightarrow V_{OH} = 5 \text{ V}$$

$$(d) \quad V_B = 7 \text{ V} \Rightarrow V_{OH} = 5 \text{ V, since } V_{DS} = 0$$

$$\text{For } v_I = V_{OH}$$

$$K_D [2(v_I - V_T)v_o - v_o^2] = K_L [V_B - v_o - V_T]^2$$

Then

$$(a) \quad (1) [2(3 - 1)V_{OL} - V_{OL}^2] = (0.4) [4 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.657 \text{ V}$$

$$(b) \quad (1) [2(4 - 1)V_{OL} - V_{OL}^2] = (0.4) [5 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.791 \text{ V}$$

$$(c) \quad (1) [2(5 - 1)V_{OL} - V_{OL}^2] = (0.4) [6 - V_{OL} - 1]^2 \Rightarrow V_{OL} = 0.935 \text{ V}$$



(d) Load in non-sat region

$$i_{DD} = i_{DL}$$

$$(1) \left[ 2(5 - 1)V_{OL} - V_{OL}^2 \right] = (0.4) \left[ 2(7 - V_{OL} - 1)(5 - V_{OL}) - (5 - V_{OL})^2 \right]$$

$$\begin{aligned} 8V_{OL} - V_{OL}^2 &= (0.4) \left[ 2(6 - V_{OL})(5 - V_{OL}) - (25 - 10V_{OL} + V_{OL}^2) \right] \\ &= (0.4) \left[ 2(30 - 11V_{OL} + V_{OL}^2) - 25 + 10V_{OL} - V_{OL}^2 \right] \\ &= (0.4) \left[ 60 - 22V_{OL} + 2V_{OL}^2 - 25 + 10V_{OL} - V_{OL}^2 \right] \end{aligned}$$

$$8V_{OL} - V_{OL}^2 = 14 - 4.8V_{OL} + 0.4V_{OL}^2$$

$$1.4V_{OL}^2 - 12.8V_{OL} + 14 = 0$$

$$V_{OL} = \frac{12.8 \pm \sqrt{163.84 - 4(1.4)(14)}}{2(1.4)}$$

$$V_{OL} = 1.27V$$

For load

$$V_{DS}(\text{sat}) = 7 - 1.27 - 1 = 4.73V$$

$$V_{DS} = 5 - 1.27 = 3.73 \text{ non-sat}$$

16.12

(a)

$$P = i_D \cdot V_{DD}$$

$$150 = i_D \cdot (3) \Rightarrow i_D = 50 \mu A$$

$$i_D = K_L (-V_{TNL})^2$$

$$50 = \left( \frac{80}{2} \right) \left( \frac{W}{L} \right)_L [ -(-1) ]^2 \Rightarrow \left( \frac{W}{L} \right)_L = 1.25$$

$$\frac{K_D}{K_L} \left[ 2(3 - 0.5)(0.1) - (0.1)^2 \right] = [ -(-1) ]^2$$

$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} = 2.04 \Rightarrow \left( \frac{W}{L} \right)_D = 2.55$$

For the Load:

$$V_{OL} = V_{DD} + V_{TNL} = 3 - 1 \Rightarrow V_{OL} = 2V$$

$$\sqrt{2.04} (V_H - 0.5) = [ -(-1) ] \Rightarrow V_H = 1.20V$$

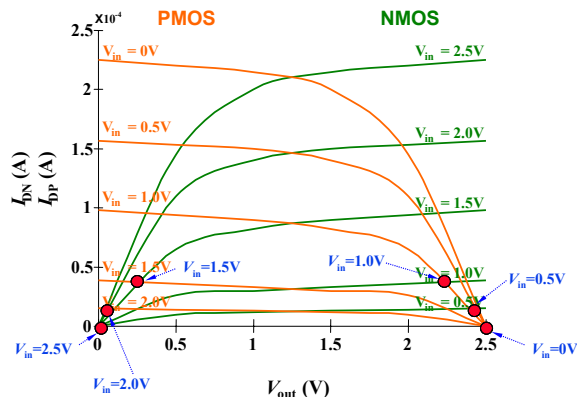
For the Driver:

$$V_{OL} = V_H - V_{TND} = 1.20 - 0.5 \Rightarrow V_{OL} = 0.70V$$

$$V_H = 1.20V$$

## DC Analysis of the CMOS Inverter

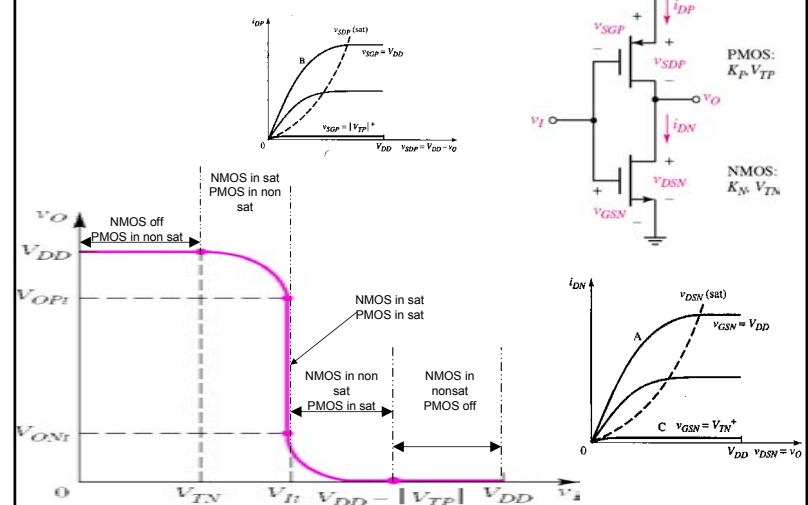
### CMOS Inverter Load Lines



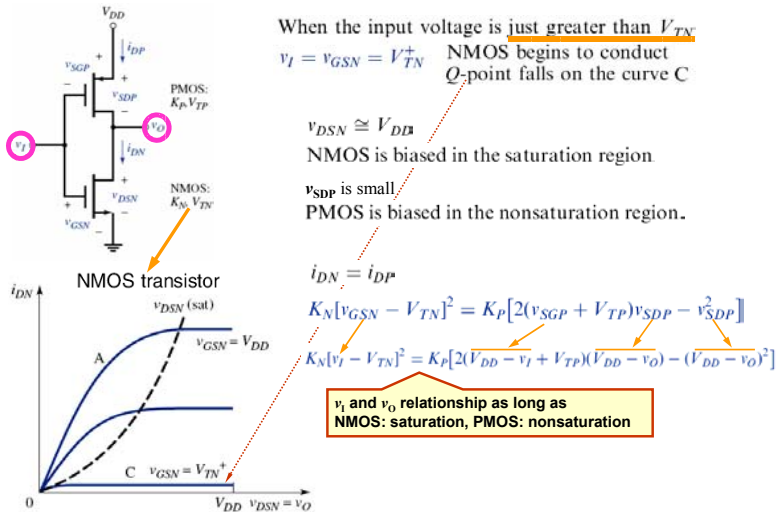
0.25um,  $W/L_n = 1.5$ ,  $W/L_p = 4.5$ ,  $V_{DD} = 2.5V$ ,  $V_{TN} = 0.4V$ ,  $V_{TP} = -0.4V$

## Complete voltage transfer characteristics,

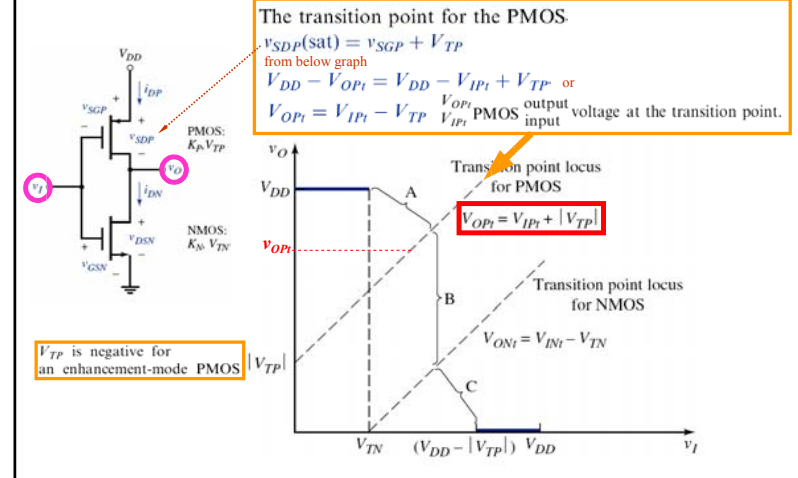
### CMOS inverter



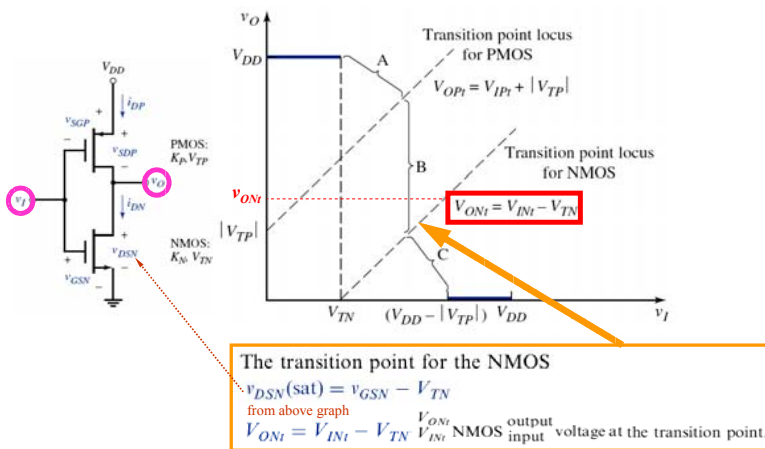
## DC Analysis of the CMOS Inverter



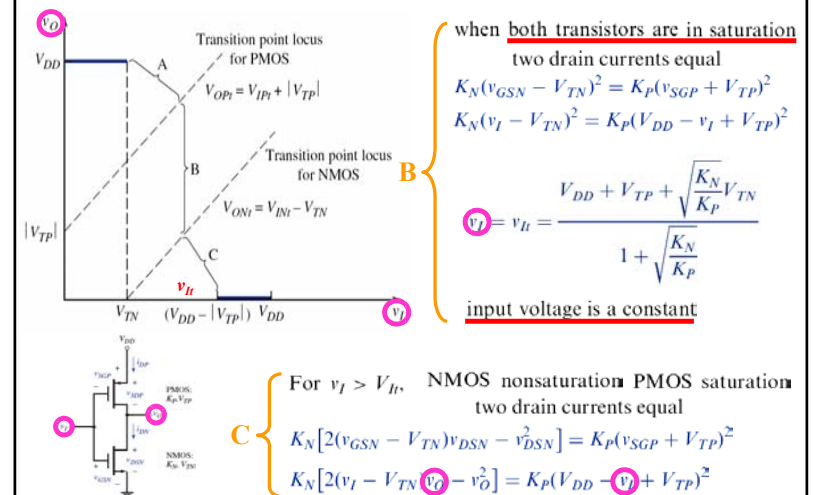
## DC Analysis of the CMOS Inverter



## DC Analysis of the CMOS Inverter

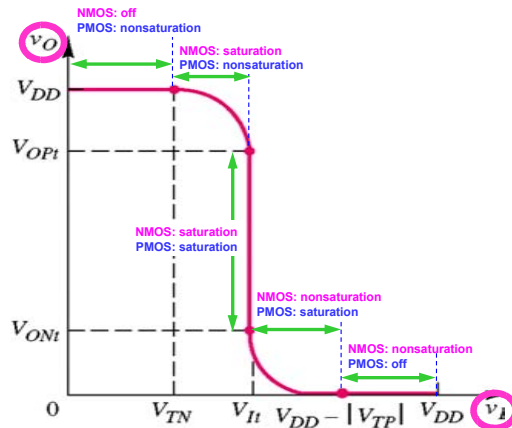


## DC Analysis of the CMOS Inverter



## DC Analysis of the CMOS Inverter

Complete voltage transfer characteristics



Example 16.9

p1041

**Example 16.9 Objective:** Determine the critical voltages on the voltage transfer curve of a CMOS inverter.

Consider a CMOS inverter biased at  $V_{DD} = 5\text{ V}$  with transistor parameters of  $K_N = K_P$  and  $V_{TN} = -V_{TP} = 1\text{ V}$ . Then consider another CMOS inverter biased at  $V_{DD} = 10\text{ V}$  with the same transistor parameters.

For  $V_{DD} = 5\text{ V}$

input voltage at the PMOS and NMOS transition points.

$$v_I = v_{It} = \frac{V_{DD} + V_{TP} + \sqrt{\frac{K_N}{K_P}} V_{TN}}{1 + \sqrt{\frac{K_N}{K_P}}} \quad v_{It} = \frac{5 + (-1) + \sqrt{1} \cdot 1}{1 + \sqrt{1}} = 2.5\text{ V}$$

output voltage at the transition point for the PMOS

$$V_{OPt} = V_{IPt} - V_{TP} \quad V_{OPt} = V_{IPt} - V_{TP} = 2.5 - (-1) = 3.5\text{ V}$$

output voltage at the transition point for the NMOS

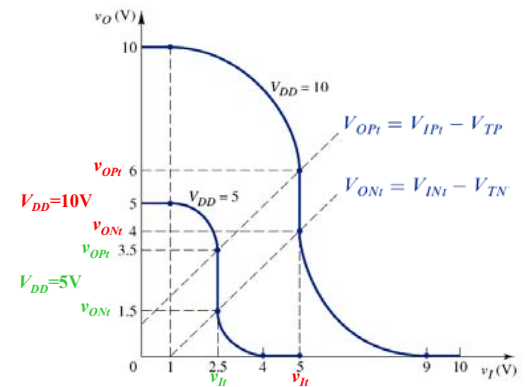
$$V_{ONt} = V_{INt} - V_{TN} \quad V_{ONt} = V_{INt} - V_{TN} = 2.5 - 1 = 1.5\text{ V}$$

For  $V_{DD} = 10\text{ V}$

$$V_{It} = 5\text{ V} \quad V_{OPt} = 6\text{ V} \quad V_{ONt} = 4\text{ V}$$

Example 16.9

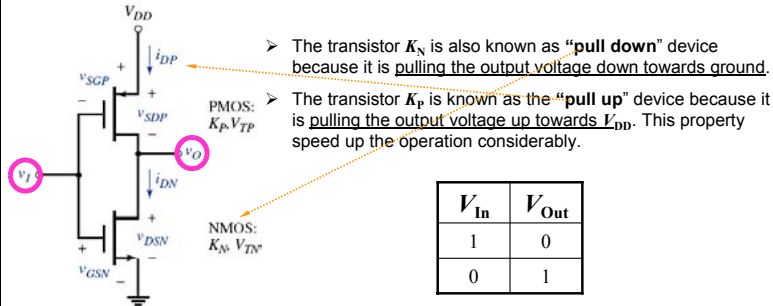
p1041



Voltage transfer characteristics, CMOS inverter biased at either  $V_{DD} = 5\text{ V}$  or  $V_{DD} = 10\text{ V}$

## DC Analysis of the CMOS Inverter

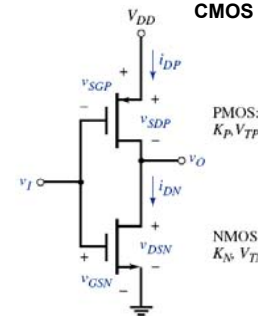
- CMOS inverter: series combination of PMOS and NMOS
- To form the **input**, gates of the two MOSFET are connected.
- To form the **output**, the drains are connected together.



The **static power dissipation** during both extreme cases (logic 1 or 0) is almost **zero** because  $i_{DP} = i_{DN} = 0$ .

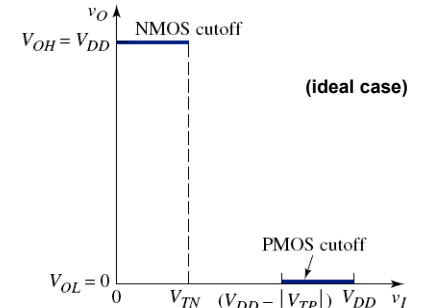
## DC Analysis of the CMOS Inverter

### CMOS Inverter in either High or Low State



Ideally, the power dissipation of the CMOS inverter is zero.

Practical device  
CMOS inverter (~ nW)  
NMOS inverter (~ mW)



CMOS inverter output voltage for input voltage in either high state or low state

## DC Analysis of the CMOS Inverter

Ideally, the current in the CMOS inverter in either steady-state condition is zero quiescent power dissipation is zero.

due to the reverse-biased pn junctions, the power dissipation may be in the nanowatt range rather than in the milliwatt range of NMOS inverters.

Without this feature, VLSI would not be possible.

## CMOS Inverter Design Consideration

- The CMOS inverter usually design to have,

$$(1) \quad V_{TN} = |V_{TP}|$$

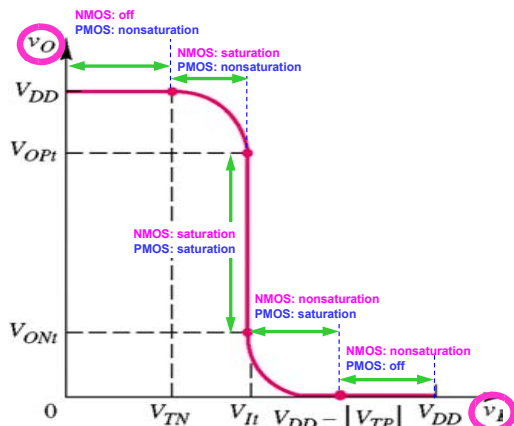
$$(2) \quad k'_N \left( \frac{W}{L} \right) = k'_P \left( \frac{W}{L} \right) \quad \text{But } k'_N > k'_P \text{ (because } \mu_N > \mu_P \text{)}$$

- How equation (2) can be satisfied ?

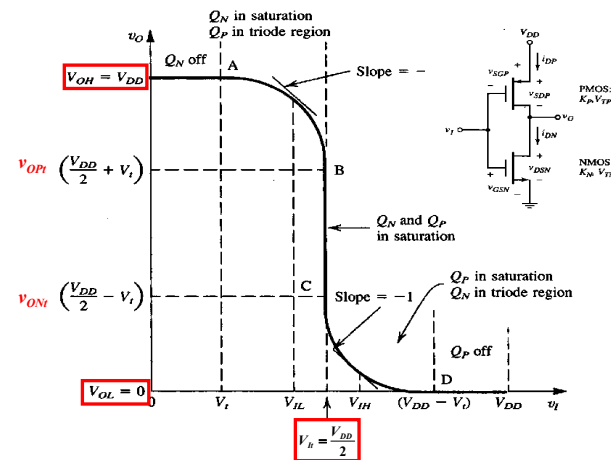
- ✓ This can achieved if **width of the PMOS** is made two or three times than that of the NMOS device.
- ✓ This is very important in order to provide a **symmetrical transition, results in wide noise margin**.

## DC Analysis of the CMOS Inverter

Complete voltage transfer characteristics



## Symmetrical Properties of the CMOS Inverter



The voltage transfer characteristic of the CMOS inverter.

p1101

For the CMOS inverter in Figure 16.34, let  $V_{TN} = +0.4 \text{ V}$ ,  $V_{TP} = -0.4 \text{ V}$ ,  $k'_n = 80 \mu\text{A/V}^2$ ,  $k'_p = 40 \mu\text{A/V}^2$ , and  $V_{DD} = 3.3 \text{ V}$ . (a) Let  $(W/L)_n = 2$  and  $(W/L)_p = 4$ . (i) Find the transition points for the p-channel and n-channel transistors.  $v_{It}$   $v_{OPt}$   $v_{ONt}$   
(ii) Find  $v_I$  when  $v_O = 0.4 \text{ V}$  and when  $v_O = 2.9 \text{ V}$ .  
(b) For  $(W/L)_n = (W/L)_p = 2$ , repeat part (a).

$$K_N = \frac{k'_n}{2} \left( \frac{W}{L} \right)$$

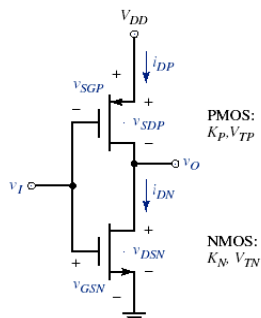


Figure 16.34 CMOS inverter

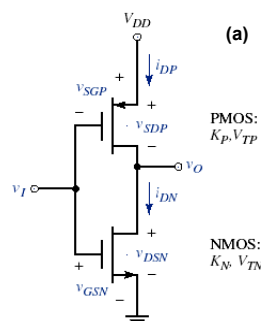


Figure 16.34 CMOS inverter

$$(a) \quad K_n = \left( \frac{80}{2} \right) (2) = 80 \mu\text{A/V}^2$$

$$K_p = \left( \frac{40}{2} \right) (4) = 80 \mu\text{A/V}^2$$

$$(i) \quad V_{It} = \frac{V_{DD} + V_{TP} + \sqrt{\frac{K_n}{K_p}} \cdot V_{TN}}{1 + \sqrt{\frac{K_n}{K_p}}} = \frac{3.3 - 0.4 + (1)(0.4)}{1 + 1} = 1.65 \text{ V}$$

$$\text{PMOS: } V_{OPt} = V_{It} - V_{TP} = 1.65 - (-0.4) \Rightarrow V_{OPt} = 2.05 \text{ V}$$

$$\text{NMOS: } V_{ONt} = V_{It} - V_{TN} = 1.65 - (0.4) \Rightarrow V_{ONt} = 1.25 \text{ V}$$

(ii) For  $v_O = 0.4 \text{ V}$ : NMOS: Non-sat; PMOS: Sat

$$K_p [2(V_{GSP} - V_{TP})V_{OS} - V_{OS}^2] = K_n [V_{GSN} + V_{TN}]^2$$

$$2(v_I - 0.4)(0.4) - (0.4)^2 = (3.3 - v_I - 0.4)^2$$

$$v_I = 1.89 \text{ V}$$

$$\text{For } v_O = 2.9 \text{ V, By symmetry}$$

$$v_I = 1.65 - (1.89 - 1.65) = 1.41 \text{ V}$$

$$(b) \quad K_n = \left(\frac{80}{2}\right)(2) = 80 \mu A/V^2$$

$$K_p = \left(\frac{40}{2}\right)(2) = 40 \mu A/V^2$$

$$K_p = \frac{k_p}{2} \left(\frac{W}{L}\right)$$

$$(i) \quad V_n = \frac{3.3 - 0.4 + \sqrt{\frac{80}{40} \cdot (0.4)}}{1 + \sqrt{\frac{80}{40}}} = 1.44 V$$

Transition points

PMOS:

$$V_{OP1} = 1.44 - (-0.4) \Rightarrow V_{\alpha} = 1.84 V$$

NMOS:

$$V_{ON1} = 1.44 - 0.4 \Rightarrow V_{\alpha} = 1.04 V$$

(ii)

For  $v_o = 0.4 V$ 

$$(80)[2(v_i - 0.4)(0.4) - (0.4)^2] = (40)[3.3 - v_i - 0.4]^2$$

$$v_i = 1.62 V$$

For  $v_o = 2.9 V$ : NMOS: Sat, PMOS: Non-sat

$$(80)[v_i - 0.4]^2 = (40)[2(3.3 - v_i - 0.4)(0.4) - (0.4)^2]$$

$$v_i = 1.16 V$$

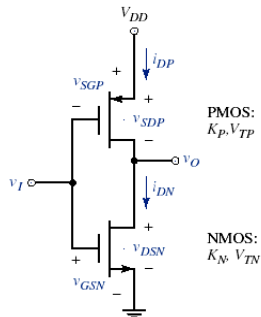
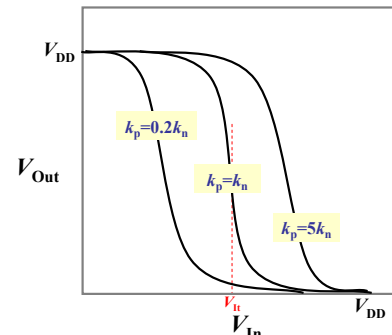


Figure 16.34 CMOS inverter

CMOS Inverter  $V_{TC}$ Increase  $W$  of PMOS→  $k_p$  increases→  $V_{It}$  moves to rightIncrease  $W$  of NMOS→  $k_n$  increases→  $V_{It}$  moves to left

$$\text{for } V_{It} = \frac{V_{DD}}{2}$$

$$\rightarrow k_N = k_P, W_N \approx W_P$$

$$v_I = v_{It} = \frac{V_{DD} + V_{TP} + \sqrt{\frac{K_N}{K_P} V_{TN}}}{1 + \sqrt{\frac{K_N}{K_P}}}$$

CMOS Inverter  $V_{TC}$ Effects of  $V_{It}$  adjustment□ Result from changing  $k_p/k_n$  ratio:

- Inverter threshold  $V_{It} \neq V_{DD}/2$
- Rise and fall delays unequal
- Noise margins not equal

## □ Reasons for changing inverter threshold:

- Want a **faster** **delay** for one type of transition (rise/fall)
- Remove **noise** from input signal: increase one noise margin at expense of the other

## Problem 16.31

**16.31** Consider the series of CMOS inverters in Figure P16.31. The threshold voltages of the n-channel transistors are  $V_{TN} = 0.8 V$ , and the threshold voltages of the p-channel transistors are  $V_{TP} = -0.8 V$ . The conduction parameters are all equal. (a) Determine the range of  $v_{O1}$  for which both  $N_1$  and  $P_1$  are biased in the saturation region. (b) If  $v_{O2} = 0.6 V$ , determine the values of  $v_{O3}$ ,  $v_{O1}$ , and  $v_I$ .

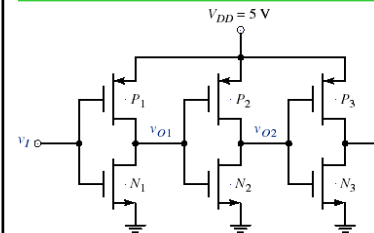


Figure P16.31

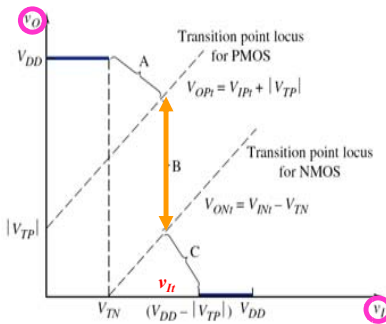
$$(a) \quad V_{ON1} \leq v_{O1} \leq V_{OP1}$$

By symmetry,  $V_{It} = 2.5 V$ 

$$V_{OP1} = 2.5 + 0.8 = 3.3 V$$

$$\text{and } V_{ON1} = 2.5 - 0.8 = 1.7 V$$

$$\text{So } \underline{1.7 \leq v_{O1} \leq 3.3 V}$$



**16.31** Consider the series of CMOS inverters in Figure P16.31. The threshold voltages of the n-channel transistors are  $V_{TN} = 0.8 \text{ V}$ , and the threshold voltages of the p-channel transistors are  $V_{TP} = -0.8 \text{ V}$ . The conduction parameters are all equal. (a) Determine the range of  $v_{O1}$  for which both  $N_1$  and  $P_1$  are biased in the saturation region. (b) If  $v_{O2} = 0.6 \text{ V}$ , determine the values of  $v_{O3}$ ,  $v_{O1}$ , and  $v_{I1}$ .

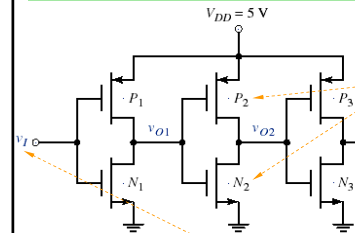


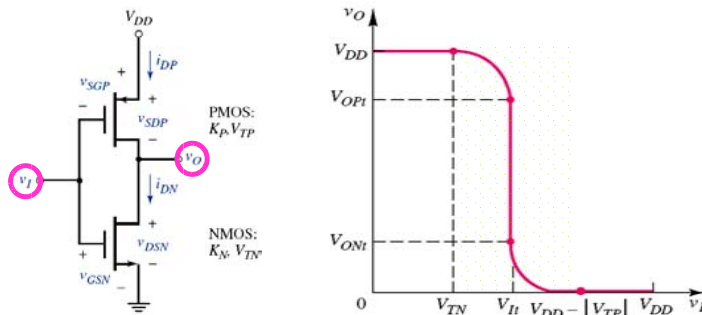
Figure P16.31

(b) For  $v_{O2} = 0.6 \text{ V} < V_{TN} \Rightarrow v_{O1} = 5 \text{ V}$   
 $N_1$  in nonsaturation and  $P_1$  in saturation. From Equation  $K_N[2(v_I - V_{TN})v_O - v_O^2] = K_P(V_{DD} - v_I + V_{TP})^2$   
 $[2(\nu_{I2} - 0.8)(0.6) - (0.6)^2] = [5 - \nu_{I2} - 0.8]^2$   
 $1.2\nu_{I2} - 1.32 = 17.64 - 8.4\nu_{I2} + \nu_{I2}^2$   
 or  
 $\nu_{I2}^2 - 9.6\nu_{I2} + 18.96 = 0$   
 So  $\nu_{I2} = \underline{v_{O1} = 2.78 \text{ V}}$   
 For  $v_{O1} = 2.78$ , both  $N_1$  and  $P_1$  in saturation. Then  
 $\underline{v_{I1} = 2.5 \text{ V}}$

## DC Analysis of the CMOS Inverter

### CMOS inverter currents

- When the output of a CMOS inverter is either at a logic 1 or 0, the current in the circuit is zero.
- When the input voltage is in the range  $V_{TN} < v_I < V_{DD} - |V_{TP}|$  both transistors are conducting and a current exists in the inverter.



## DC Analysis of the CMOS Inverter

### CMOS inverter currents

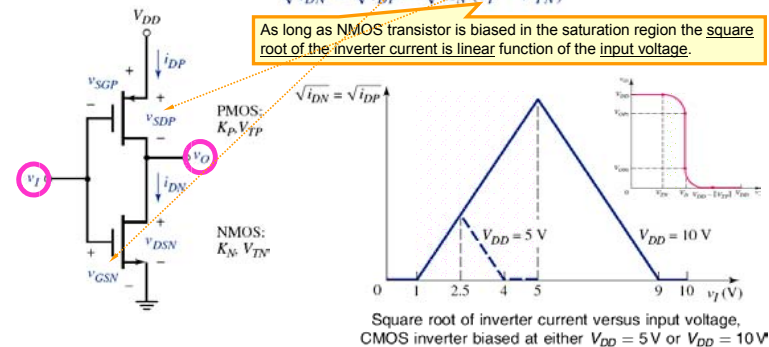
- ❑ When **NMOS** transistor is biased in the **saturation** region

➤ The current in the inverter is controlled by  $v_{GSN}$  and the PMOS  $v_{SDP}$  adjusts such that  $i_{DP} = i_{DN}$ .

$$i_{DN} = i_{DP} = K_N(v_{GSN} - V_{TN})^2 = K_N(v_I - V_{TN})^2$$

$$\sqrt{i_{DN}} = \sqrt{i_{DP}} = \sqrt{K_N}(v_I - V_{TN})$$

As long as NMOS transistor is biased in the saturation region the square root of the inverter current is linear function of the input voltage.





## DC Analysis of the CMOS Inverter

### CMOS inverter currents

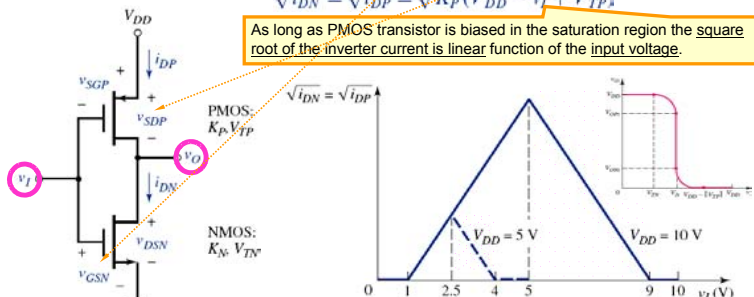
#### □ When PMOS transistor is biased in the saturation region

- The current in the inverter is controlled by  $v_{SGP}$  and the NMOS  $v_{DSN}$  adjusts such that  $i_{DP} = i_{DN}$ .

$$i_{DN} = i_{DP} = K_P(V_{DD} - v_I + V_{TP})^2$$

$$\sqrt{i_{DN}} = \sqrt{i_{DP}} = \sqrt{K_P}(V_{DD} - v_I + V_{TP})$$

As long as PMOS transistor is biased in the saturation region the square root of the inverter current is linear function of the input voltage.



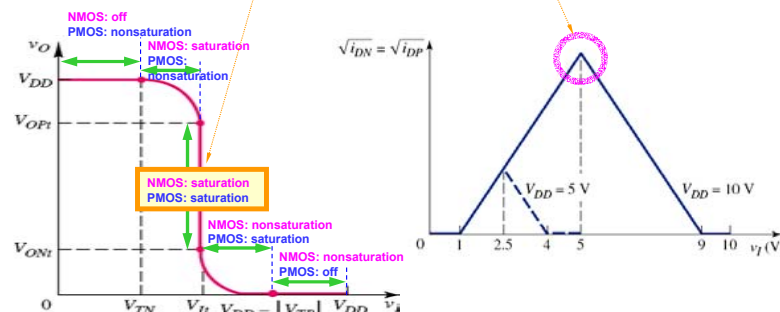
Square root of inverter current versus input voltage, CMOS inverter biased at either  $V_{DD} = 5V$  or  $V_{DD} = 10V$

## DC Analysis of the CMOS Inverter

### CMOS inverter currents

At the inverter switching point, both transistors are biased in the saturation region and both transistors influence the current.

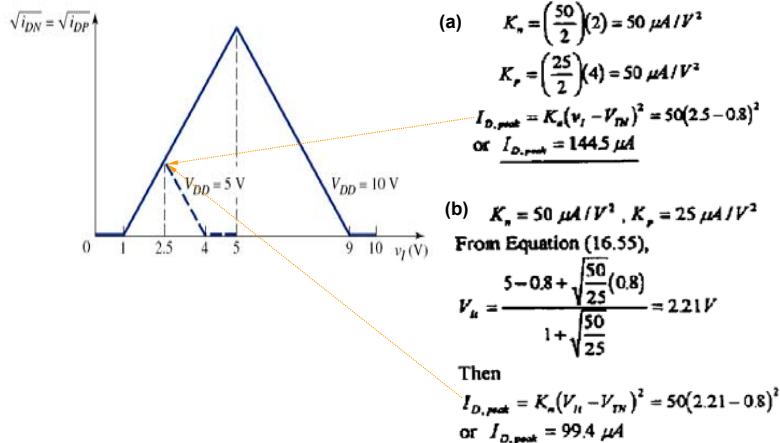
- the actual current characteristic does not have a sharp discontinuity in the slope.



### Problem 16.33

p1102

**16.33** The transistor parameters in the CMOS inverter are:  $k_n' = 50 \mu A/V^2$ ,  $k_p' = 25 \mu A/V^2$ ,  $V_{TN} = 0.8 V$ , and  $V_{TP} = -0.8 V$ . (a) For  $(W/L)_n = 2$  and  $(W/L)_p = 4$ , determine the peak current in the inverter during a switching cycle for  $V_{DD} = 5 V$ . (b) Repeat part (a) for  $(W/L)_n = (W/L)_p = 2$ .

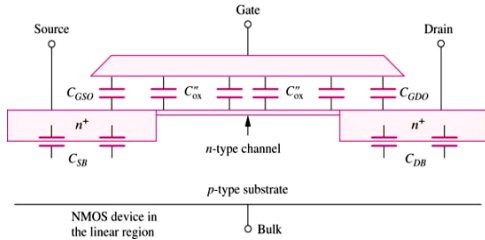


## Power Dissipation

- There is **no power dissipation in the CMOS inverter** when the output is either at logic 0 or 1. However, **during switching** of the CMOS inverter from low logic 0 to logic 1, **current flows** and **power is dissipated**.
- Usually CMOS inverter and logic circuit are used to drive other MOS devices by connecting a capacitor across the output of a CMOS inverter. This **capacitor** must be **charged** and **discharged** during the switching cycle.

## NMOS Transistor Capacitances

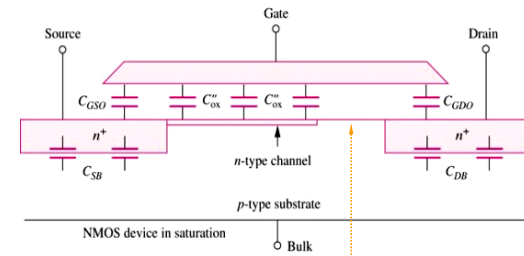
### Triode Region



$C_{ox}$  = Gate-Channel capacitance per unit area ( $F/m^2$ )  
 $C_{GC}$  = Total gate channel capacitance  
 $C_{GS}$  = Gate-Source capacitance  
 $C_{GD}$  = Gate-Drain capacitance  
 $C_{GSO}$  and  $C_{GDO}$  = overlap capacitances ( $F/m$ )

## NMOS Transistor Capacitances

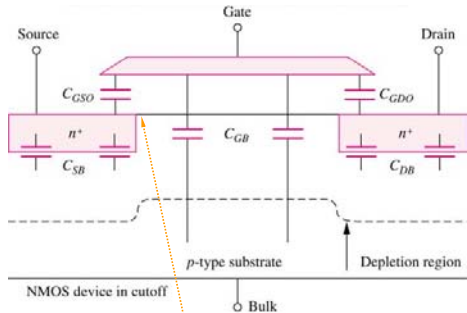
### Saturation Region



➤ Drain is no longer connected to channel.

## NMOS Transistor Capacitances

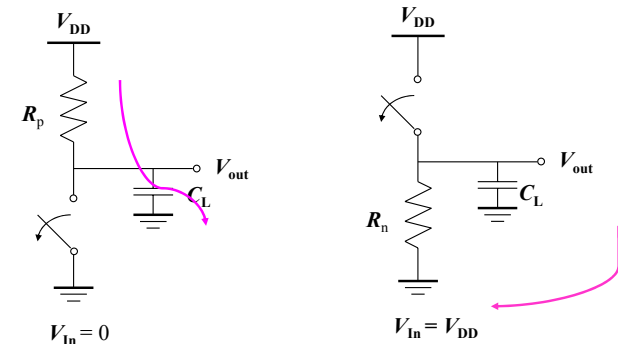
### Cutoff Region



➤ Conducting channel region is completely gone.  
 $C_{GB}$  = Gate-Bulk capacitance  
 $C_{GBO}$  = Gate-Bulk capacitance per unit width.

## CMOS Inverter

### Switch Model of Dynamic Behavior



➤ Gate response time is determined by the time to charge  $C_L$  through  $R_p$  (discharge  $C_L$  through  $R_n$ )

## CMOS Inverter Power

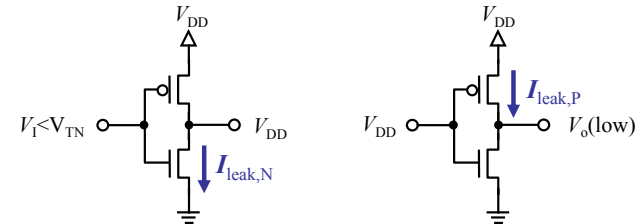
❑ Power has three components

- **Static power**: when input isn't switching
- **Dynamic capacitive power**: due to charging and discharging of load capacitance
- **Dynamic short-circuit power**: direct current from  $V_{DD}$  to  $G_{nd}$  when both transistors are on

## CMOS Inverter Power

### Static Power Consumption

- ❖ Static current: in CMOS there is no static current as long as  $V_{in} < V_{TN}$  or  $V_{in} > V_{DD} + V_{TP}$
- ❖ **Leakage current**: determined by "off" transistor
- ❖ Influenced by transistor width, supply voltage, transistor threshold voltages



## CMOS Inverter Power

### Dynamic Capacitive Power and Energy stored in the PMOS

**Case I: When the input is at logic 0**

PMOS is conducting and NMOS is in cutoff mode and the load capacitor must be charged through the PMOS device.

**Power dissipation** in the PMOS transistor;

$$P_P = i_L V_{SDP} = i_L (V_{DD} - V_O)$$

The current and output voltages are related by,

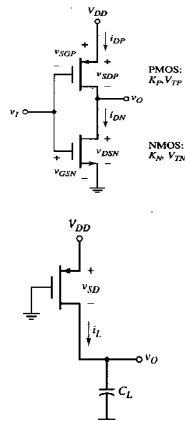
$$i_L = C_L dv_O/dt$$

Similarly the **energy dissipation** in the PMOS device

$$E_P = \int_0^{V_{DD}} P_P dv_O = \int_0^{V_{DD}} i_L (V_{DD} - v_O) dv_O = \int_0^{V_{DD}} C_L (V_{DD} - v_O) dv_O = C_L V_{DD} \int_0^{V_{DD}} (V_{DD} - v_O) dv_O = C_L V_{DD} \left[ V_{DD} v_O - \frac{v_O^2}{2} \right]_0^{V_{DD}} = C_L V_{DD} \left( V_{DD}^2 - \frac{V_{DD}^2}{2} \right) = \frac{1}{2} C_L V_{DD}^2$$

$$E_P = C_L V_{DD}^2 \left[ \frac{v_O}{V_{DD}} - \frac{v_O^2}{2 V_{DD}^2} \right]_0^{V_{DD}} = C_L V_{DD}^2 \left( 1 - \frac{1}{2} \right) = \frac{1}{2} C_L V_{DD}^2$$

$E_P = \frac{1}{2} C_L V_{DD}^2$  the energy stored in the capacitor  $C_L$  when the output is high.



## CMOS Inverter Power

### Dynamic Capacitive Power and Energy stored in the PMOS

**Case II: when the input is high and out put is low:**

During switching all the energy stored in the load capacitor is dissipated in the NMOS device because NMOS is conducting and PMOS is in cutoff mode.

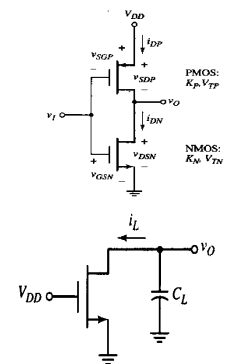
The energy dissipated in the NMOS inverter;

$$E_N = \frac{1}{2} C_L V_{DD}^2$$

The total energy dissipated during one switching cycle  $E_T = E_P + E_N = \frac{1}{2} C_L V_{DD}^2 + \frac{1}{2} C_L V_{DD}^2 = C_L V_{DD}^2$

$$E_T = P \cdot t \rightarrow P = \frac{E_T}{t} \rightarrow P = f E_T \rightarrow f C_L V_{DD}^2$$

This implied that the **power dissipation** in the CMOS inverter is directly proportional to switching frequency and  $V_{DD}^2$



## CMOS Inverter Power

### Dynamic Capacitive Power

$$P_{dyn} = C_L V_{DD}^2 f$$

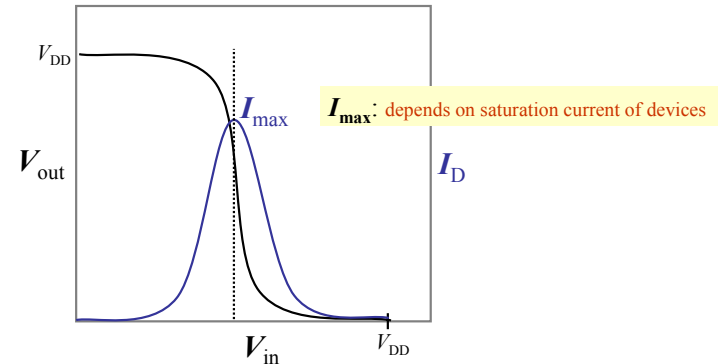
Formula for dynamic power

- Does not (directly) depend on device sizes
- Does not depend on switching delay
- Applies to general CMOS gate in which:
  - Switched capacitances are lumped into  $C_L$
  - Output swings from GND to  $V_{DD}$
  - Input signal approximated as step function
  - Gate switches with frequency  $f$

## CMOS Inverter Power

### Dynamic Short-Circuit Power

- Short-circuit current flows from  $V_{DD}$  to GND when both transistors are on saturation mode.



## Inverter Power Consumption

### Total Power Consumption

$$P_{tot} = P_{dyn} + P_{sc} + P_{stat}$$

$$P_{tot} = C_L V_{DD}^2 f + V_{DD} I_{max} \left( \frac{t_r + t_f}{2} \right) f + V_{DD} I_{leak}$$

$$P_{tot} \sim C_L V_{DD}^2 f$$

## Power Reduction

### Reducing dynamic capacitive power

- **Lower the voltage!!**
  - ❖ Quadratic effect on dynamic power
- **Reduce capacitance!!**
  - ❖ Short interconnect lengths
  - ❖ Drive small gate load (small gates, small fan-out)
- **Reduce frequency!!**
  - ❖ Lower clock frequency
  - ❖ Lower signal activity

$$P_{dyn} = C_L V_{DD}^2 f$$

## Power Reduction

### ❑ Reducing short-circuit current

- Fast rise/fall times on input signal
- Reduce input capacitance
- Insert small buffers to "clean up" slow input signals before sending to large gate

### ❑ Reducing leakage current

- Small transistors (leakage proportional to width)
- Lower voltage

Example 16.10 on Page 1045

Given that for a CMOS inverter

$$C_L = 2 \text{ pF}$$

$$V_{DD} = 5 \text{ V}$$

$$f = 100 \text{ KHz}$$

calculate the power dissipation in the CMOS inverter

Sol:

AS we know relation between power, frequency and capacitance

$$P = f C_L V_{DD}^2$$

$$\Rightarrow P = (10^5)(2 \times 10^{-12})(5)^2$$

$$P = 5 \times 10^{-6} \text{ W}$$

Comments,

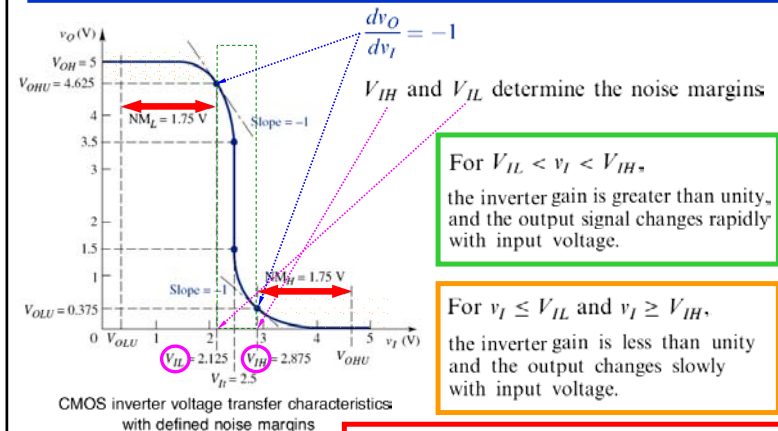
The typical values of static power dissipation in NMOS inverter is on the order of 500 mW. The power dissipation of the CMOS inverter is

in the above example is 500 times smaller than NMOS inverter.

⇒ CMOS is ideal for practical applications as (in integrated digital circuits).

The power dissipation, in static values

## Concept of Noise Margins



$$NM_L = V_{IL} - V_{OLU} \quad \text{Noise Margin for low input}$$

$$NM_H = V_{OHU} - V_{IH} \quad \text{Noise Margin for high input}$$

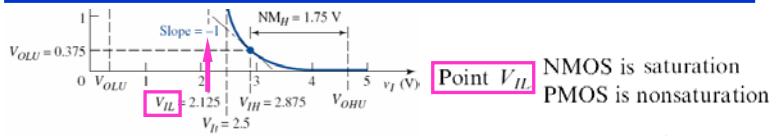
## Chapter 16

# MOSFET Digital Circuits

### Chapter 16.3.4

## CMOS Inverter Noise Margin

## CMOS Inverter Noise Margins



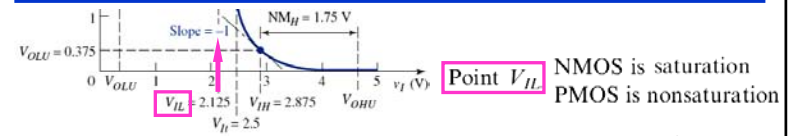
The relationship between the input and output voltages of CMOS when NMOS is saturation PMOS is nonsaturation

$$i_{DN} = i_{DP}$$

$$K_N[v_{GSN} - V_{TN}]^2 = K_P[2(v_{SGP} + V_{TP})v_{SDP} - v_{SDP}^2]$$

$$K_N[v_I - V_{TN}]^2 = K_P[2(V_{DD} - v_I + V_{TP})(V_{DD} - v_O) - (V_{DD} - v_O)^2]$$

## CMOS Inverter Noise Margins



The relationship between the input and output voltages of CMOS when NMOS is saturation PMOS is nonsaturation

$$K_N[v_I - V_{TN}]^2 = K_P[2(V_{DD} - v_I + V_{TP})(V_{DD} - v_O) - (V_{DD} - v_O)^2]$$

Taking the derivative with respect to  $v_I$

$$2K_N[v_I - V_{TN}] = K_P[-2(V_{DD} - v_O) - 2(V_{DD} - v_I + V_{TP})\frac{dv_O}{dv_I} - 2(V_{DD} - v_O)(-\frac{dv_O}{dv_I})]$$

$$\frac{dv_O}{dv_I} = -1 \rightarrow K_N[v_I - V_{TN}] = -K_P[(V_{DD} - v_O) - (V_{DD} - v_I + V_{TP}) + (V_{DD} - v_O)]$$

$$v_O = V_{OHU} = \frac{1}{2} \left\{ \left( 1 + \frac{K_N}{K_P} \right) v_I + V_{DD} - \left( \frac{K_N}{K_P} \right) V_{TN} - V_{TP} \right\}$$

$$v_I = V_{IL} = V_{TN} + \frac{(V_{DD} + V_{TP} - V_{TN})}{\left( \frac{K_N}{K_P} - 1 \right)} \left[ 2 \sqrt{\frac{\frac{K_N}{K_P}}{\frac{K_N}{K_P} + 3}} - 1 \right]$$

## CMOS Inverter Noise Margins

$$v_O = V_{OHU} = \frac{1}{2} \left\{ \left( 1 + \frac{K_N}{K_P} \right) v_I + V_{DD} - \left( \frac{K_N}{K_P} \right) V_{TN} - V_{TP} \right\}$$

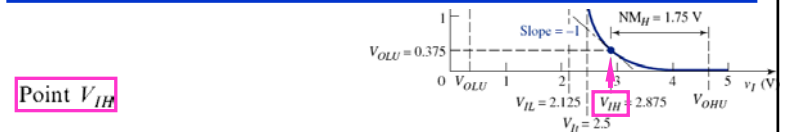
If CMOS is symmetrical,  $K_N = K_P$

$$v_O = V_{OHU}(K_N=K_P) = \frac{1}{2} [2v_I + V_{DD} - V_{TN} - V_{TP}]$$

$$K_N[v_I - V_{TN}]^2 = K_P[2(V_{DD} - v_I + V_{TP})(V_{DD} - v_O) - (V_{DD} - v_O)^2]$$

$$v_I = V_{IL}(K_N=K_P) = V_{TN} + \frac{3}{8} (V_{DD} + V_{TP} - V_{TN}) \quad \text{for } K_N = K_P$$

## CMOS Inverter Noise Margins



The relationship between the input and output voltages of CMOS when NMOS is nonsaturation PMOS is saturation

$$K_N[2(v_I - V_{TN})v_O - v_O^2] = K_P(V_{DD} - v_I + V_{TP})^2$$

Taking the derivative with respect to  $v_I$

$$K_N[2(v_I - V_{TN})\frac{dv_O}{dv_I} + 2v_O - 2v_O\frac{dv_O}{dv_I}] = 2K_P(V_{DD} - v_I + V_{TP})(-1)$$

$$\frac{dv_O}{dv_I} = -1 \rightarrow K_N[-(v_I - V_{TN}) + v_O + v_O] = -K_P[V_{DD} - v_I + V_{TP}]$$

$$v_O = V_{OLU} = \frac{v_I \left( 1 + \frac{K_N}{K_P} \right) - V_{DD} - \left( \frac{K_N}{K_P} \right) V_{TN} - V_{TP}}{2 \left( \frac{K_N}{K_P} \right)}$$

$$v_I = V_{IH} = V_{TN} + \frac{(V_{DD} + V_{TP} - V_{TN})}{\left( \frac{K_N}{K_P} - 1 \right)} \left[ \frac{2 \frac{K_N}{K_P}}{\sqrt{3 \frac{K_N}{K_P} + 1}} - 1 \right]$$

## CMOS Inverter Noise Margins

$$v_O = V_{OLU} = \frac{v_I \left(1 + \frac{K_N}{K_P}\right) - V_{DD} - \left(\frac{K_N}{K_P}\right) V_{TN} - V_{TP}}{2 \left(\frac{K_N}{K_P}\right)}$$

If CMOS is symmetrical,  $K_N = K_P$

$$v_O = V_{OLU(K_N=K_P)} = \frac{1}{2} \{2v_I - V_{DD} - V_{TN} - V_{TP}\}$$

$$K_N [2(v_I - V_{TN})v_O - v_O^2] = K_P (V_{DD} - v_I + V_{TP})^2$$

$$v_I = V_{IH(K_N=K_P)} = V_{TN} + \frac{5}{8} (V_{DD} + V_{TP} - V_{TN}) \quad \text{for } K_N = K_P$$

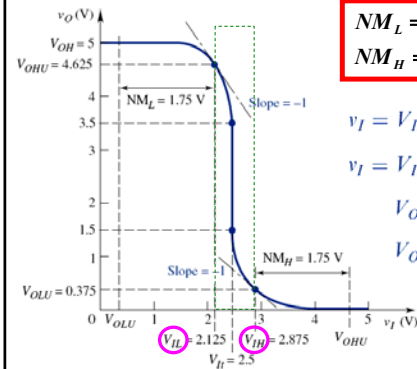
## CMOS Inverter Noise Margins



### Summary Noise Margin of a Symmetrical CMOS Inverter

$$NM_L = V_{IL} - V_{OLU} \quad \text{Noise Margin for low input}$$

$$NM_H = V_{OHU} - V_{IH} \quad \text{Noise Margin for high input}$$



CMOS inverter voltage transfer characteristics with defined noise margins

### Example 16.11

P1047

**Example 16.11 Objective:** Determine the noise margins of a CMOS inverter.

Consider a CMOS inverter biased at  $V_{DD} = 5\text{ V}$ . Assume the transistors are matched with  $K_N = K_P$  and  $V_{TN} = -V_{TP} = 1\text{ V}$ .

$$v_I = V_{IL(K_N=K_P)} = V_{TN} + \frac{3}{8} (V_{DD} + V_{TP} - V_{TN})$$

$$V_{IL} = V_{TN} + \frac{3}{8} (V_{DD} + V_{TP} - V_{TN}) = 1 + \frac{3}{8} (5 - 1 - 1) = 2.125\text{ V}$$

$$v_I = V_{IH(K_N=K_P)} = V_{TN} + \frac{5}{8} (V_{DD} + V_{TP} - V_{TN})$$

$$V_{IH} = V_{TN} + \frac{5}{8} (V_{DD} + V_{TP} - V_{TN}) = 1 + \frac{5}{8} (5 - 1 - 1) = 2.875\text{ V}$$

$$V_{OHU(K_N=K_P)} = \frac{1}{2} [2v_I + V_{DD} - V_{TN} - V_{TP}] - V_{OLU(K_N=K_P)} = \frac{1}{2} [2v_I - V_{DD} - V_{TN} - V_{TP}]$$

$$V_{OHU} = \frac{1}{2} [2V_{IL} + V_{DD} - V_{TN} - V_{TP}] = \frac{1}{2} [2(2.125) + 5 - 1 + 1] = 4.625\text{ V}$$

$$V_{OLU} = \frac{1}{2} [2V_{IH} - V_{DD} - V_{TN} - V_{TP}] = \frac{1}{2} [2(2.875) - 5 - 1 + 1] = 0.375\text{ V}$$

noise margins

$$NM_H = V_{OHU} - V_{IH} = 4.625 - 2.875 = 1.75\text{ V}$$

$$NM_L = V_{IL} - V_{OLU} = 2.125 - 0.375 = 1.75\text{ V}$$

## Chapter 16

# MOSFET Digital Circuits

### Chapter 16.4

## CMOS Logic Circuits



## CMOS Logic Circuits

- Large scale integrated CMOS logic circuits including watches, calculators, and microprocessors are constructed by using basic CMOS NOR and NAND gates.

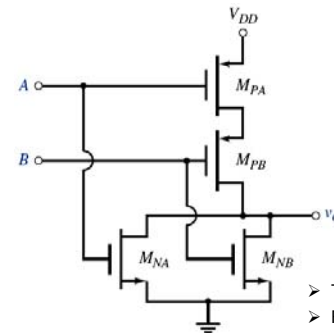
Therefore, understanding of these basic gates is very important for the designing of very large scale integrated (VLSI) logic circuits.

### CMOS NOR and NAND Gates

## CMOS Logic Circuits

### CMOS NOR and NAND Gates

- ❑ CMOS **NOR gate** can be constructed by using **two parallel NMOS** devices and **two series PMOS** transistors.



truth table

$\frac{A}{0}$	$\frac{B}{0}$	$\frac{v_O}{V_{DD}}$
$V_{DD}$	0	0
0	$V_{DD}$	0
$V_{DD}$	$V_{DD}$	0

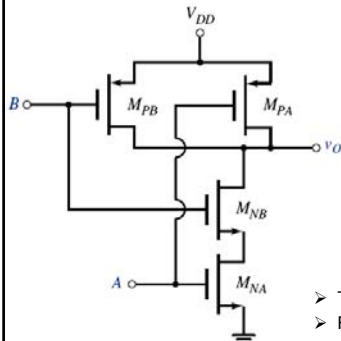
- The output is at **logic 1** when **all inputs are low**.
- For **all other** possible inputs, output is low or at **logic 0**.

Two-input CMOS NOR logic circuit

## CMOS Logic Circuits

### CMOS NOR and NAND Gates

- ❑ CMOS **NAND gate** can be constructed by using **two parallel PMOS** devices and **two series NMOS** transistors.



truth table

$\frac{A}{0}$	$\frac{B}{0}$	$\frac{v_O}{V_{DD}}$
0	0	$V_{DD}$
$V_{DD}$	0	$V_{DD}$
0	$V_{DD}$	$V_{DD}$
$V_{DD}$	$V_{DD}$	0

- The is at **logic 0** when **all inputs are high**.
- For **all other** possible inputs, output is high or at **logic 1**.

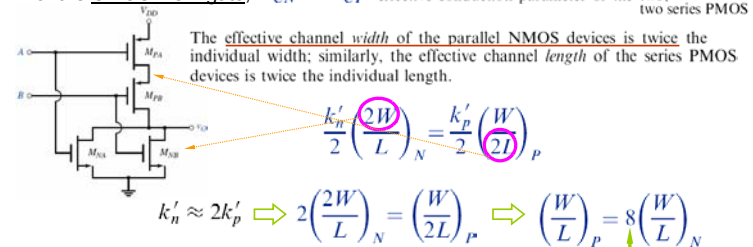
Two-input CMOS NAND logic circuit

## CMOS Logic Circuits

### How can we design CMOS NOR symmetrical gate?

To obtained symmetrical switching times for the high-to-low and low-to-high output transitions, the effective conduction (design) parameters of the **composite PMOS** and **composite NMOS** device must be equal.

For the **CMOS NOR gate**,  $K_{CN} = K_{CP}$  effective conduction parameter of the twoparallel NMOS two series PMOS

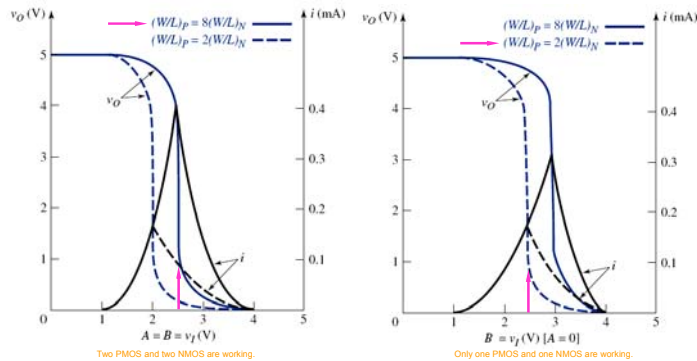


In order to get the symmetrical switching properties, the width to length ratio of **PMOS** transistor must be approximately eight times that of the NMOS device.

For asymmetrical case, switching time is longer!!

## CMOS Logic Circuits

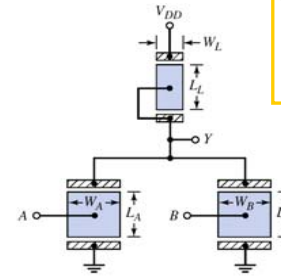
How can we design CMOS NOR symmetrical gate?



Voltage transfer characteristics, two-input CMOS NOR logic circuit for various width-to-length ratios

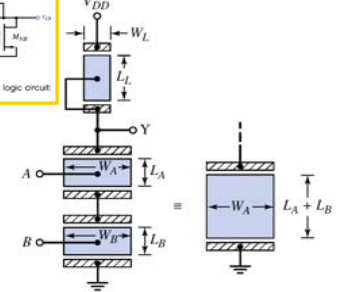
## Concept of Effective Width to Length Ratios

Parallel combination



For the **NOR gate** the **effective width of the drivers transistors doubles**.  
The effective aspect ratio is increased.

Series combination



For the **NAND gate** the **effective length of the driver transistors doubles**.  
The effective aspect ratio is decreased.

**D16.19** In the two-input CMOS NAND gate in Figure 16.45(a), determine the relationship between the  $(W/L)$  ratios of the n-channel and p-channel transistors, such that composite conduction parameters of the PMOS and NMOS devices are equal. (Ans.  $(W/L)_N = 2(W/L)_P$ )

**D16.20** Design a three-input CMOS NOR logic gate such that the effective conduction parameters of the composite PMOS and NMOS transistors are equal. Determine  $(W/L)_P/(W/L)_N$ , where  $(W/L)$  is the width-to-length ratio of the individual PMOS and NMOS transistors. (Ans.  $(W/L)_P = 18(W/L)_N$ )

D16.19 on page 1051

Reference: 2-input CMOS NAND gate.

Determine relationship b/w the n-channel and p-channel transistors, such that composite conduction parameters of the PMOS and NMOS devices are equal.  
ie  $K_{N, eff} = K_{P, eff}$

Sol: Inspection of figure 16.45a shows that the effective channel width of the parallel PMOS device is twice the individual width and effective channel length of the series NMOS transistors is twice the individual length i.e.

$$\left(\frac{K_N}{2}\right)\left(\frac{W}{L}\right)_N = \left(\frac{K_P}{2}\right)\left(\frac{2W}{L}\right)_P$$

$$\Rightarrow \left(\frac{2K_P}{2}\right)\left(\frac{W}{L}\right)_N = \left(\frac{K_P}{2}\right)\left(\frac{2W}{L}\right)_P$$

$$\Rightarrow \frac{1}{2}\left(\frac{W}{L}\right)_N = K_P\left(\frac{W}{L}\right)_P$$

$$\Rightarrow \left(\frac{W}{L}\right)_N = 2\left(\frac{W}{L}\right)_P$$

16.20 on page 1051

Design a three input CMOS NOR logic gate

Such that

$$K_{N, eff} = K_{P, eff}$$

$$\text{Determine } (W/L)_P / (W/L)_N$$

Sol: in case of the input CMOS NOR gate

we have three NMOS parallel transistors

three PMOS series transistor

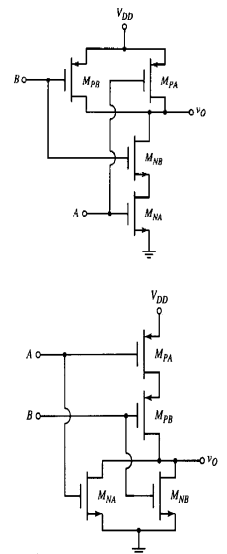
$$\Rightarrow \left(\frac{K_N}{3}\right)\left(\frac{3W}{L}\right)_N = \left(\frac{K_P}{3}\right)\left(\frac{W}{L}\right)_P$$

$$\text{Since } K_N \approx 2K_P$$

$$\left(\frac{2K_P}{3}\right)\left(\frac{3W}{L}\right)_N = \left(\frac{K_P}{3}\right)\left(\frac{W}{L}\right)_P$$

$$3\left(\frac{W}{L}\right)_N = \frac{1}{6}\left(\frac{W}{L}\right)_P$$

$$\Rightarrow \frac{(W/L)_P}{(W/L)_N} = 18$$



## CMOS Logic Circuits

### Fan-In and Fan-Out

- **Fan-in** of a gate is the number of its inputs.  
Thus a four input NOR gate has a fan-In of 4.
- **Fan-Out** is the maximum number of load gates that may be connected to the output.

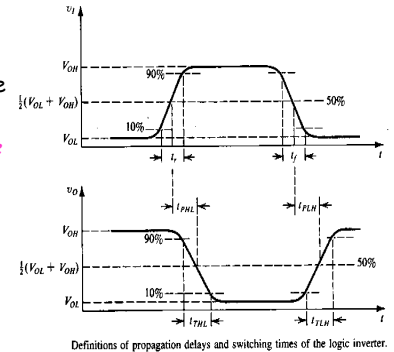
Since the CMOS logic gate will be driving other CMOS logic gates, the quiescent current required to drive the other CMOS gates is essentially zero.  
➡ the maximum fanout is virtually limitless.

However,

- ❖ Each additional load gate increases the load capacitance they must be charge and discharge as the driver gate changes state. This places a practical limit on the maximum allowable number of load gates.

## Switching Time and Propagation Delay Time

- The dynamic performance of a logic circuit family is characterized by propagation delay of its basic inverter. The propagation delay time is defined as the *average of low-to-high propagation delay time and the high-to-low propagation delay time*.
- The propagation delay time is directly proportional to the **switching time** and increases as the Fan-out increases. Therefore, the maximum Fan-out is limited by the maximum acceptable propagation delay time.

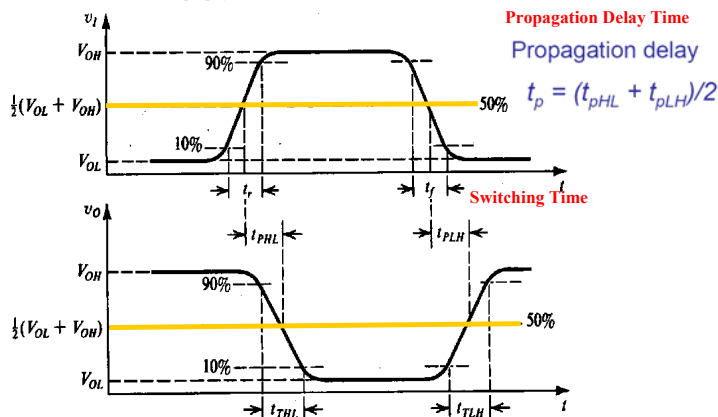


Each additional load gate increases the load capacitance they must be charge and discharge as the driver gate changes state. This places a practical limit on the maximum allowable number of load gates.

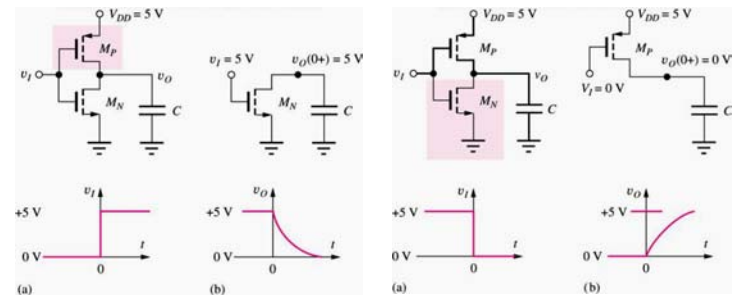
## CMOS Logic Circuits

### Switching Time and Propagation Delay Time

Definitions of propagation delays and switching times of the logic inverter.

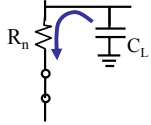


## Propagation Delay Estimate



- The two modes of capacitive charging/discharging that contribute to propagation delay

## Switch-level model



Delay estimation using switch-level model (for general RC circuit):

$$I = C \frac{dV}{dt} \rightarrow dt = \frac{C}{I} dV$$

$$I = \frac{V}{R} \rightarrow dt = \frac{RC}{V} dV$$

$$t_1 - t_0 = t_p = \int_{V_0}^{V_1} \frac{RC}{V} dV$$

$$t_p = RC [\ln(V_1) - \ln(V_0)] = RC \ln\left(\frac{V_1}{V_0}\right)$$

## Switch-level model

- For fall delay  $t_{phl}$ ,  $V_0 = V_{cc}$ ,  $V_1 = V_{cc}/2$

$$t_p = RC \ln\left(\frac{V_1}{V_0}\right) = RC \ln\left(\frac{\frac{1}{2}V_{cc}}{V_{cc}}\right)$$

$$t_p = RC \ln(0.5)$$

$$t_{phl} = 0.69 R_n C_L$$

$$t_{plh} = 0.69 R_p C_L$$

Standard RC-delay equations

## Chapter 16

# MOSFET Digital Circuits

## Chapter 16.6

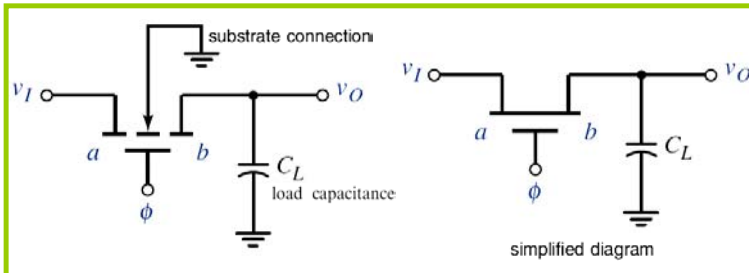
## Transmission Gates

## Transmission Gates

- Use of transistors as **switches** between driving circuits and load circuits are called **transmission gates** because switches can transmit information from one circuit to another.
- NMOS and CMOS transmission gate.

## NMOS Transmission Gate

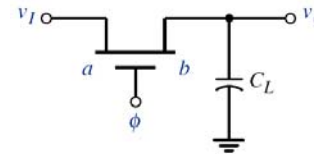
- The bias applied to the transistor determines which terminal acts as the **drain** and which terminal acts as the **source**.



- $C_L$  load capacitance input gate capacitance of a MOS logic circuit.
- transistor must be bilateral  
must be able to conduct current in either direction.  
This is a natural feature of MOSFETs.

## NMOS Transmission Gate

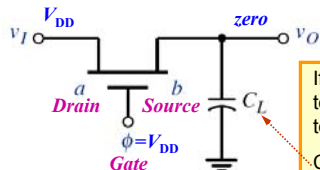
### As an Open Switch



When gate voltage  $\phi=0$ , the n-channel transistor is **cut-off** and the transistor acts as an **open switch**

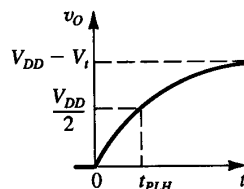
## NMOS Transmission Gate

### As an Open Switch



@ High input

If  $\phi = V_{DD}$ ,  $v_I = V_{DD}$ , and  $v_O$  is initially ( $t=0$ ) zero, terminal **a** acts as the **drain** since its bias is  $V_{DD}$ . terminal **b** acts as the **source** since its bias is zero. Current enters the drain from the input **charging** up the **capacitor**.



As  $C_L$  charges up and  $V_O$  increases, the gate to source voltage  $V_{GS}$  become equal to threshold voltage  $V_{TN}$ , the capacitance stop charging and current goes to zero.

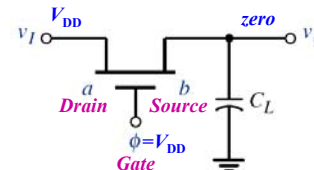
This implies that the  $V_O = V_O(\max)$  when  $V_{GS} = V_{TN}$

Or  
 $V_O(\max) = V_{DD} - V_{TN}$

This implies that output voltage never will be equal to  $V_{DD}$ ; rather it will be lower by  $V_{TN}$ .  
This is one of the disadvantage of an NMOS transmission gate when  $V_I = \text{high}$

## NMOS Transmission Gate

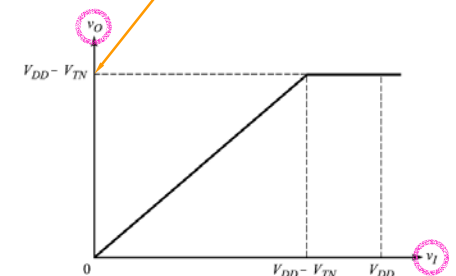
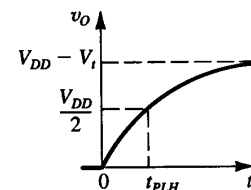
### As an Open Switch



@ High input

$$v_{GS} = \phi - v_O = V_{DD} - v_O$$

$$v_O(\max) = V_{DD} - V_{TN}$$



Output voltage versus input voltage characteristics

## Characteristics of NMOS transmission gate (at low input)

- When  $V_I=0$  and  $\phi=V_{DD}$  and  $V_O=V_{DD}-V_{TN}$  at  $t=0$  (initially). It is to be noted that in the present case terminal b acts as the drain and terminal a acts as the source.
- Under these conditions the gate to source voltage is,

$$V_{GS} = \phi - V_I$$

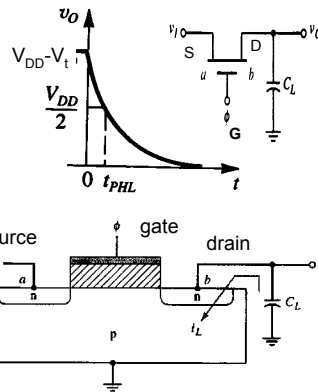
$$V_{GS} = V_{DD} - 0$$

$$V_{GS} = V_{DD}$$

This implies that value of  $V_{GS}$  is constant. In this case the capacitor is fully discharge to zero as the drain current goes to zero.

$$V_O = 0$$

This implies that the NMOS transistor provide a "good" logic 0 when  $V_I$  is low

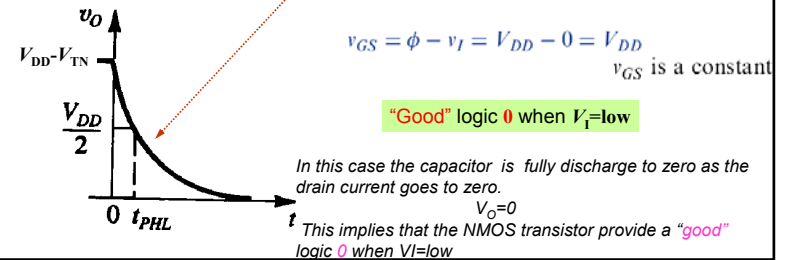


## NMOS Transmission Gate

### As an Open Switch

@ Low input

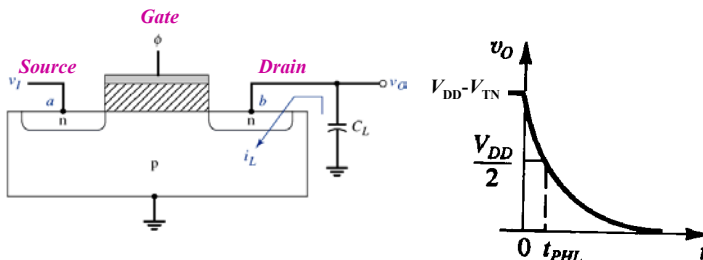
When  $\phi = V_{DD}$ ,  $v_I = 0$ , and  $v_O = V_{DD} - V_{TN}$ , at  $t=0$ , terminal a acts as the **source** since its bias is **zero**. terminal b acts as the **drain** since its bias is **high**. **Capacitor discharges** as current enters the drain. Stop discharging drain current goes **zero**.



## NMOS Transmission Gate

□ Why NMOS transmission gate does not remain in a static condition?

- The **reverse leakage current** due to reverse bias between terminal b and ground begins to **discharge the capacitor**, and the circuit does not remain in a static condition.

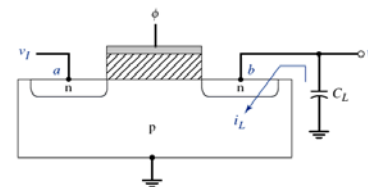


### Example 16.13

p1060

**Example 16.13 Objective:** Estimate the rate at which the output voltage  $v_O$  in Figure 16.57 decreases with time.

Assume the capacitor is initially charged to  $v_O = 4\text{ V}$ . Let  $C_L = 1\text{ pF}$  and assume the reverse-biased pn junction leakage current is a constant at  $i_L = 1\text{ nA}$ .



The voltage across the capacitor

$$v_O = -\frac{1}{C_L} \int i_L dt = -\frac{i_L}{C_L} t + K_1$$

minus sign indicates that the current is leaving the positive terminal of the capacitor.

$K_1 = v_O(t=0) = 4\text{ V}$  initial condition

$$v_O = 4 - \frac{i_L}{C_L} t$$

The rate at which the output voltage decreases

$$\frac{dv_O}{dt} = -\frac{i_L}{C_L} = -\frac{10^{-9}}{10^{-12}} = -1000\text{ V/s} \Rightarrow -1\text{ V/ms}$$

the capacitor would completely discharge in 4 ms.

**Example 16.14 Objective:** Determine the output of an NMOS inverter driven by a series of NMOS transmission gates.

Consider the circuit shown in Figure 16.58. The NMOS inverter is driven by three NMOS transmission gates in series. Assume the threshold voltages of the n-channel transmission gate transistors and the driver transistor are  $V_{TN} = +0.8 \text{ V}$ , and the threshold voltage of the load transistor is  $V_{TNL} = -1.5 \text{ V}$ . Let  $K_D/K_L = 3$  for the inverter. Determine  $v_O$  for  $v_I = 0$  and  $v_I = 5 \text{ V}$ .

**Solution:** The three NMOS transmission gates in series act as an AND/NAND function. If  $v_I = 0$  and  $A = B = C = \text{logic } 1 = 5 \text{ V}$ , the gate capacitance to driver  $M_D$  becomes completely discharged, which means that  $v_{O1} = v_{O2} = v_O = 0$ . Driver  $M_D$  is cut off and  $v_O = 5 \text{ V}$ .

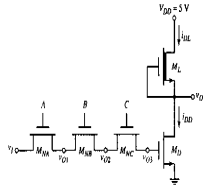


Figure 16.58 NMOS inverter driven by three NMOS transmission gates in series

If  $v_I = 5 \text{ V}$  and  $A = B = C = \text{logic } 1 = 5 \text{ V}$ , the three transmission gates are biased in their conducting state, and the gate capacitance of  $M_D$  becomes charged. For transistor  $M_{A,L}$ , the current becomes zero when the gate-to-source voltage is equal to the threshold voltage, or, from Equation (16.87(b)),

$$v_{O1} = V_{DD} - V_{TN} = 5 - 0.8 = 4.2 \text{ V}$$

Transistors  $M_{B,L}$  and  $M_{C,L}$  also cut off when the gate-to-source voltages are equal to the threshold voltage; therefore,

$$v_{O2} = v_{O1} = V_{DD} - V_{TN} = 5 - 0.8 = 4.2 \text{ V}$$

This result shows that the drain-to-source voltages of  $M_{B,L}$  and  $M_{C,L}$  are also zero. A threshold voltage drop is lost in the first transmission gate, but additional threshold voltage drops are not lost in subsequent NMOS transmission gates in series.

For a voltage of  $v_{O1} = 4.2 \text{ V}$  applied to the gate of  $M_D$ , the driver is biased in the nonsaturation region and the load is biased in the saturation region. From  $i_{DD} = i_{DL}$ , we have

$$K_D[2(v_{O1} - V_{TN})v_O - v_O^2] = K_L[-V_{TNL}]^2$$

The output voltage is found to be

$$v_O = 0.112 \text{ V}$$

If any one of the transmission gate voltages,  $A$  or  $B$  or  $C$ , switches to a logic 0, then  $v_{O1}$  will begin to discharge through a reverse-biased pn junction in the transmission gates, which means that  $v_O$  will increase with time.

**Comment:** In this example, the inverter is again in a dynamic condition; that is, when any transmission gate is cut off, the output voltage changes with time. However, this type of circuit can be used in clocked digital systems.

## Test Your Understanding

**16.26** The threshold voltage of the NMOS transmission gate transistor in Figure 16.55(a) is  $V_{TN} = 1 \text{ V}$ . Determine the quiescent output voltage  $v_O$  for: (a)  $v_I = \phi = 5 \text{ V}$ ; (b)  $v_I = 3 \text{ V}$ ,  $\phi = 5 \text{ V}$ ; (c)  $v_I = 4.2 \text{ V}$ ,  $\phi = 5 \text{ V}$ ; and (d)  $v_I = 5 \text{ V}$ ,  $\phi = 3 \text{ V}$ . (Ans. (a)  $v_O = 4 \text{ V}$  (b)  $v_O = 3 \text{ V}$  (c)  $v_O = 4 \text{ V}$  (d)  $v_O = 2 \text{ V}$ )

16.26 on page 1062.

Given that

$$V_{TN} = 1 \text{ V}$$

Determine  $v_O$  for

$$\textcircled{a} \quad v_I = \phi = 5 \text{ V}$$

$$\textcircled{b} \quad v_I = 3 \text{ V}, \phi = 5 \text{ V}$$

$$\textcircled{c} \quad v_I = 4.2 \text{ V}, \phi = 5 \text{ V}$$

Sol:

$$\textcircled{a} \quad \text{When } v_I = \phi$$

$$v_I = 5 \text{ V}$$

$$\phi = 5 \text{ V}$$

$\Rightarrow$  The transmission gate

is biased in conducting state

and the gate capacitance ~~of~~ ~~the~~ ~~transistor~~

becomes fully charged when  $v_{GS} = V_{TN}$

$\Rightarrow$  Equation 16.87b can be used

$$\Rightarrow v_O = V_{DD} - V_{TN} \quad V_{DD} = \phi$$

$$\Rightarrow v_O = 5 - 1 = 4 \text{ V}$$

$$v_O = 4 \text{ V}$$

$$\textcircled{b} \quad \text{When } v_I = 3 \text{ V}$$

$$\phi = 5 \text{ V}$$

$\Rightarrow$  As we know in NMOS

transmission gate when

$v_I \leq V_{DD} - V_{TN}$ , the

upper output follows input i.e.

$$v_O = v_I$$

~~Since~~ ~~Since~~ ~~Since~~  $v_I \leq 4.2$

In given case

$$\Rightarrow v_O = 3 \text{ V}$$

$$\textcircled{c} \quad \text{When}$$

$$v_I = 4.2 \text{ V}$$

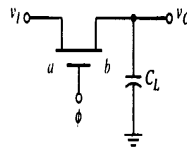
$$\phi = 5 \text{ V}$$

In this case

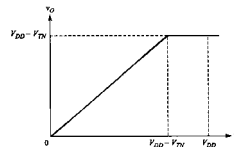
$$v_I = V_{DD} - V_{TN}$$

$$\Rightarrow v_I = 5 - 1 = 4 \text{ V}$$

$$v_I = 4 \text{ V}$$



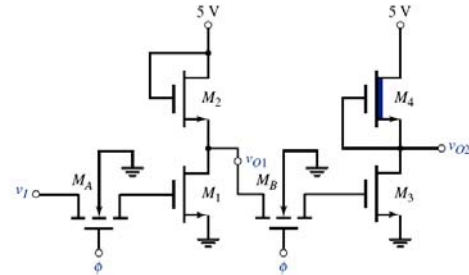
$$v_{O(max)} = V_{DD} - V_{TN}$$



Example 16.52

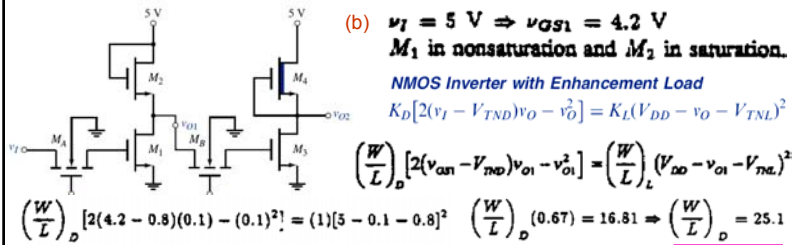
p1106

**D16.52** For the circuit in Figure P16.52, the input voltage  $v_I$  is either  $0.1 \text{ V}$  or  $5 \text{ V}$ . Let  $\phi = 5 \text{ V}$ . The threshold voltages are  $V_{TN} = -1.5 \text{ V}$  for  $M_4$  and  $V_{TN} = 0.8 \text{ V}$  for all other transistors. The width-to-length ratios are 1 for  $M_2$  and  $M_4$  and 10 for  $M_A$  and  $M_B$ . (a) What are the logic 1 values of  $v_{O1}$  and  $v_{O2}$ ? (b) Design the width-to-length ratios of  $M_1$  and  $M_3$  such that the logic 0 values of  $v_{O1}$  and  $v_{O2}$  are  $0.1 \text{ V}$ . **Neglect the body effect.**



$$\text{(a)} \quad v_{O1}(\text{logic } 1) = 4.2 \text{ V}, \quad v_{O2}(\text{logic } 1) = 5 \text{ V}$$





Now  $v_{O1} = 4.2\text{ V} \Rightarrow v_{GS3} = 4.2\text{ V}$

$M_3$  in nonsaturation and  $M_4$  in saturation.

NMOS Inverter with Depletion Load

$$\frac{K_D}{K_L} [2(v_I - V_{TND})v_{O2} - v_{O2}^2] = (-V_{TNL})^2$$

$$\left(\frac{W}{L}\right)_D [2(v_{GS3} - V_{TND})v_{O2} - v_{O2}^2] = \left(\frac{W}{L}\right)_L [-V_{TNL}]^2$$

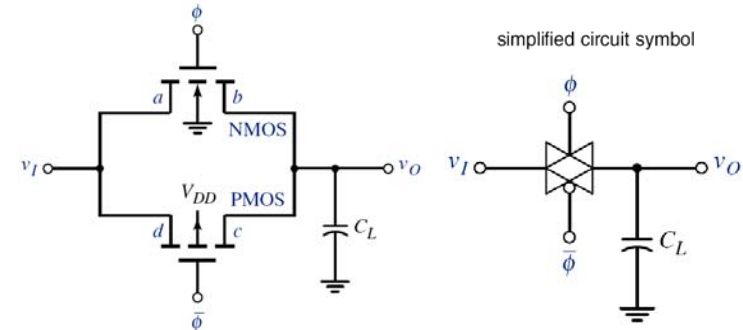
$$\left(\frac{W}{L}\right)_D [2(4.2 - 0.8)(0.1) - (0.1)^2] = (2)[-(-1.5)]^2$$

$$\left(\frac{W}{L}\right)_D (0.67) = 2.25$$

$$\left(\frac{W}{L}\right)_D = 3.36$$

## CMOS Transmission Gate

- A CMOS transmission gate can be constructed by parallel combination of NMOS and PMOS transistors, with complementary gate signals.
- The main advantage of the CMOS transmission gate compared to NMOS transmission gate is to allow the input signal to be transmitted to the output **without the threshold voltage attenuation**.



## CMOS Transmission Gate

### Case I: Input High condition

If  $\phi = V_{DD}$ ,  $\bar{\phi} = 0$ ,  $v_I = V_{DD}$ , and  $v_O$  is initially zero,

NMOS terminal  $a$  acts as the drain  
 terminal  $b$  acts as the source

PMOS terminal  $c$  acts as the drain  
 terminal  $d$  acts as the source

In order to charge the load capacitor, current enters the NMOS drain and the PMOS source.

NMOS  $v_{GSN} = \phi - v_O = V_{DD} - v_O$

$V_{GSN}$  continuously change

PMOS  $v_{SGP} = v_I - \bar{\phi} = V_{DD} - 0 = V_{DD}$

$V_{SGP}$  remains constant

when  $v_O = V_{DD} - V_{TN}$ , the NMOS cuts off and  $i_{DN} = 0$  since  $v_{GSN} = V_{TN}$ .

PMOS continues to conduct since  $v_{SGP} = V_{DD}$

In PMOS,  $i_{DP} = 0$ , when  $v_{SDP} = 0$ , which would be possible only, if,  $V_O = V_I = 5\text{ V}$

logic '1' is unattenuated

## CMOS Transmission Gate

### Case II: Input Low condition

if  $\phi = V_{DD}$ ,  $\bar{\phi} = 0$ ,  $v_I = 0$ , and  $v_O = V_{DD}$  initially

NMOS terminal  $a$  acts as the source  
 terminal  $b$  acts as the drain

PMOS terminal  $c$  acts as the source  
 terminal  $d$  acts as the drain

In order to discharge the load capacitor, current enters the NMOS drain and the PMOS source.

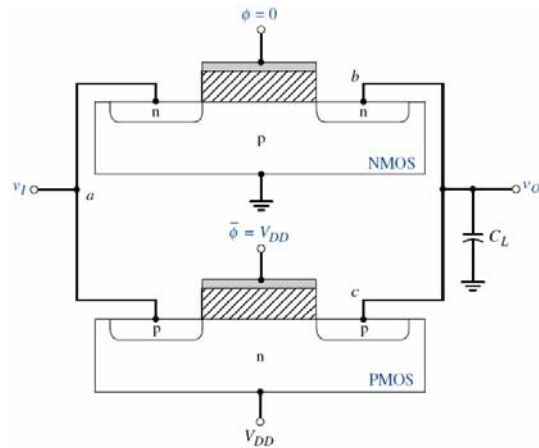
NMOS  $v_{GSN} = \phi - v_I = V_{DD} - 0 = V_{DD}$

PMOS  $v_{SGP} = v_O - \bar{\phi} = v_O - 0 = v_O$

When  $v_{SGP} = v_O = |V_{TP}|$ , the PMOS device cuts off and  $i_{DP}$  goes to zero. However, since  $v_{GSN} = V_{DD}$ , the NMOS transistor continues conducting and capacitor  $C_L$  completely discharges to zero.

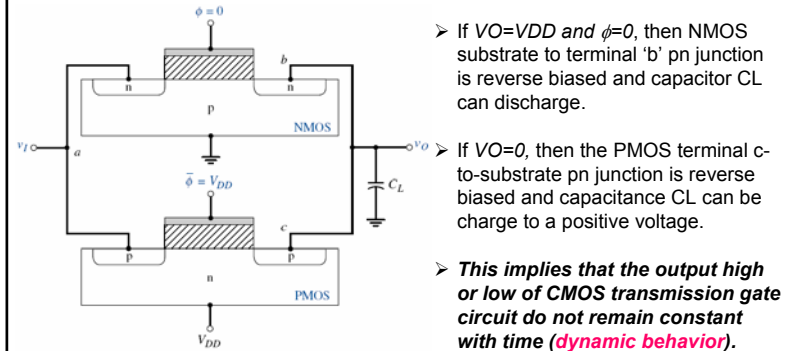
Finally,  $V_O = 0$ , which is a good logic 0.  $v_{SGP} = v_O - \bar{\phi} = v_O - 0 = v_O$

## CMOS Transmission Gate



## CMOS Transmission Gate

CMOS transmission gate remains in a dynamic condition.



### Test your understanding

p1067

**16.29** Consider the CMOS transmission gate in Figure 16.64(a). Assume transistor parameters of  $V_{TN} = +0.8\text{ V}$  and  $V_{TP} = -1.2\text{ V}$ . When  $\phi = 5\text{ V}$ , input  $v_I$  varies with time as  $v_I = 0.5t\text{ V}$  for  $0 \leq t \leq 10\text{ s}$ . Let  $v_O(t=0) = 0$  and assume  $C_L = 1\text{ pF}$ . Determine the range of times that the NMOS and PMOS devices are conducting or cut off.

### Exercise 16.29 (a)

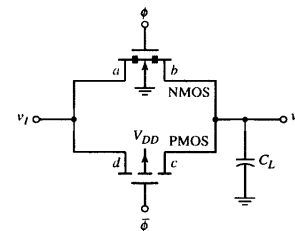
p10XX

Given that  $V_{TN} = 0.8\text{ V}$ ,  $V_{TP} = -1.2\text{ V}$ . When  $\phi = 5\text{ V}$ , input  $v_I$  varies with time as  $v_I = 0.5t$  for  $0 \leq t \leq 10\text{ sec}$ . Let  $V_O = 0$  and  $C_L = 1\text{ pF}$ . Determine the range of the times that the NMOS and PMOS devices are conducting or cutoff.

NMOS conducting for  $0 \leq v_I \leq 4.2\text{ V}$

NMOS conducting for  $0 \leq t \leq 8.4\text{ sec}$

NMOS cutoff for  $8.4 \leq t \leq 10\text{ sec}$



# HW solution

## EX16.8

$$\begin{aligned} \text{(a)} \quad V_R &= \frac{V_{DD}}{2} = \frac{2.1}{2} = 1.05 \text{ V} \\ V_{OP1} &= V_R - V_{TD} = 1.05 - (-0.4) = 1.45 \text{ V} \\ V_{ON1} &= V_R - V_{TN} = 1.05 - 0.4 = 0.65 \text{ V} \\ \text{(b)} \quad V_R &= \frac{2.1 + (-0.4) + \sqrt{0.5}(0.4)}{1 + \sqrt{0.5}} = 1.16 \text{ V} \\ V_{OP1} &= 1.16 + 0.4 = 1.56 \text{ V} \\ V_{ON1} &= 1.16 - 0.4 = 0.76 \text{ V} \\ \text{(c)} \quad V_R &= \frac{2.1 + (-0.4) + \sqrt{2}(0.4)}{1 + \sqrt{2}} = 0.938 \text{ V} \\ V_{OP1} &= 0.938 + 0.4 = 1.338 \text{ V} \\ V_{ON1} &= 0.538 \text{ V} \end{aligned}$$

## TYU16.3

$$\begin{aligned} P &= i_D \cdot V_{DD} \Rightarrow i_D = \frac{800}{5} = 160 \text{ } \mu\text{A} \\ i_D &= 160 = \frac{35}{2} \cdot \left(\frac{W}{L}\right)_L (1.4)^2 \Rightarrow \left(\frac{W}{L}\right)_L = 4.66 \\ i_D &= 160 \text{ } \mu\text{A} = \frac{35}{2} \cdot \frac{1}{3} \cdot \left(\frac{W}{L}\right)_D \left[ 2(5 - 0.8)(0.12) - (0.12)^2 \right] \Rightarrow \left(\frac{W}{L}\right)_D = 27.6 \end{aligned}$$

## TYU16.4

a. From the load transistor:

$$I_{DL} = \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_L (V_{GS} - V_{TN})^2 = \frac{35}{2} (0.5)(5 - 0.15 - 0.7)^2$$

or

$$I_{DL} = 150.7 \text{ } \mu\text{A}$$

Maximum  $v_o$  occurs when either  $A$  or  $B$  is high and  $C$  is high. For the two NMOS is series, the effective

$k'_n$  is cut in half, so

$$I_{DL} = \frac{1}{2} \left[ \left(\frac{k'_n}{2}\right) \left(\frac{W}{L}\right)_D \right] \left[ 2(V_{GSD} - V_{TND})V_{DS} - V_{DS}^2 \right]$$

or

$$150.7 = \frac{1}{2} \left[ \frac{35}{2} \cdot \left(\frac{W}{L}\right)_D \right] \left[ 2(5 - 0.7)(0.15) - (0.15)^2 \right]$$

which yields

$$\left(\frac{W}{L}\right)_D = 13.6$$

$$\text{b. } P = i_D \cdot V_{DD} = (150.7)(5) \Rightarrow P = 753 \text{ } \mu\text{W}$$

## 16.36

$$\text{a. } V_{ON1} \leq v_{o1} \leq V_{OP1}$$

By symmetry,  $V_R = 2.5 \text{ V}$

$$V_{OP1} = 2.5 + 0.8 = 3.3 \text{ V}$$

$$\text{and } V_{ON1} = 2.5 - 0.8 = 1.7 \text{ V}$$

$$\text{So } 1.7 \leq v_{o1} \leq 3.3 \text{ V}$$

$$\text{b. For } v_{O2} = 0.6 < V_{TN} \Rightarrow v_{O3} = 5 \text{ V}$$

$N_2$  in nonsaturation and  $P_2$  in saturation. From Equation (16.57),

$$\left[ 2(v_{I2} - 0.8)(0.6) - (0.6)^2 \right] = [5 - v_{I2} - 0.8]^2$$

$$1.2v_{I2} - 1.32 = 17.64 - 8.4v_{I2} + v_{I2}^2$$

or

$$v_{I2}^2 - 9.6v_{I2} + 18.96 = 0$$

$$\text{So } v_{I2} = v_{O1} = 2.78 \text{ V}$$

For  $v_{O1} = 2.78$ , both  $N_1$  and  $P_1$  in saturation. Then

$$\underline{v_I = 2.5 \text{ V}}$$

16.39  
(a) Switching Voltage, Eq. (16.43)

$$v_D = \frac{3.3 - 0.4 + \sqrt{\frac{2(4)}{8}}(0.4)}{1 + \sqrt{\frac{2(4)}{8}}} = 1.65 \text{ V} = v_{th}$$

$$i_{D, \text{peak}} = \left(\frac{80}{2}\right)(4)(1.65 - 0.4)^2 \Rightarrow i_{D, \text{peak}} = 250 \text{ } \mu\text{A}$$

(b)

$$v_D = \frac{3.3 - 0.4 + \sqrt{\frac{2(4)}{4}}(0.4)}{1 + \sqrt{\frac{2(4)}{4}}} = 1.436 \text{ V} = v_{th}$$

$$i_{D, \text{peak}} = \left(\frac{80}{2}\right)(4)(1.436 - 0.4)^2 \Rightarrow i_{D, \text{peak}} = 172 \text{ } \mu\text{A}$$

(c)

$$v_D = \frac{3.3 - 0.4 + \sqrt{\frac{2(4)}{12}}(0.4)}{1 + \sqrt{\frac{2(4)}{12}}} \Rightarrow v_{th} = 1.776 \text{ V}$$

$$i_{D, \text{peak}} = \left(\frac{80}{2}\right)(4)(1.776 - 0.4)^2 \Rightarrow i_{D, \text{peak}} = 303 \text{ } \mu\text{A}$$

## Chapter 16

# MOSFET Digital Circuits

## Chapter 16.7

## Sequential Logic Circuit

## SEQUENTIAL LOGIC CIRCUITS

In the logic circuits that we have considered in the previous sections, such as NOR and NAND logic gates, the output is determined only by the instantaneous values of the input signals. These circuits are therefore classified as combinational logic circuits.

Another class of circuits is called **sequential logic circuits**. The output depends not only on the inputs, but also on the previous history of its inputs.

This feature gives sequential circuits the property of memory. Shift registers and flip-flops are typical examples of such circuits. We will also briefly consider a full-adder circuit. The characteristic of these circuits is that they store information for a short time until the information is transferred to another part of the system.

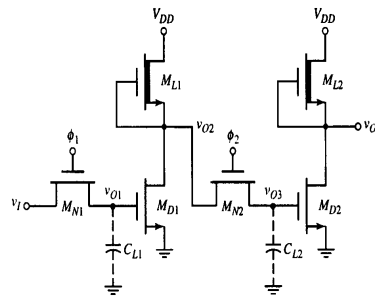
## SEQUENTIAL LOGIC CIRCUITS

- The logic circuits considered thus far are called **combinational logic circuits**. Their output depend only on the present value of input. This implies that these circuit do not have **memory**.
- Another class of the logic circuit that incorporate memory are called **sequential logic circuits**; that is, their output depend not only the present value of the input, but also on the **previous history of inputs**. **Shift registers** and **flip-flops** are typical examples of such circuits.

# SEQUENTIAL LOGIC CIRCUITS

## NMOS Dynamic Shift Registers

- A shift register can be constructed by the combination of transmission gates and inverters.
- If  $V_i = V_{DD}$  and  $\phi_i = V_{DD}$ , then a logic 1 =  $V_{DD} - V_{TN}$  would exist at  $V_{O1}$ .
- The  $C_L$  charges through  $M_{N1}$ . As  $V_{O1}$  goes high,  $V_{O2}$  goes low. If  $\phi_2$  is high low will be transmitted through  $M_{N2}$  and  $V_{O4}$  would be at logic 1. Thus logic 1 shifted from input to output.

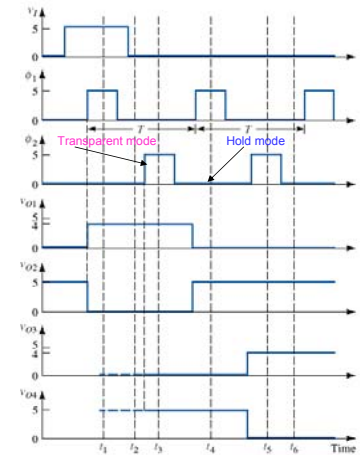
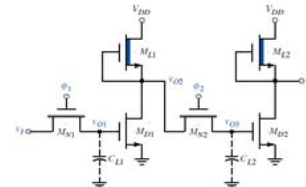


In shift register the input signal is transmitted, or shifted, from the input to the output during one clock cycle.

## Dynamic Shift Registers at Various Time

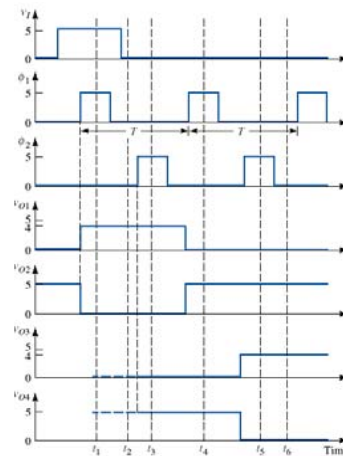
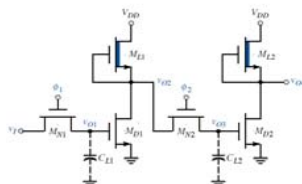
Suppose  $V_{DD}=5V$  and  $V_{TN}=1V$ .  
At  $t=t_1$ ,  $V_i=\phi_1=5V$ ,  $v_{O2}$  goes low  
At this time  $M_{N2}$  is still in cutoff ( $\phi_2=0$ )

even though input of  $M_{N2}$  has been changed. This implies that  $v_{O3}$  and  $v_{O4}$  depend on the **previous history**.  
Similarly at  $t=t_3$ ,  $\phi_2$  is high, and logic 0 at  $v_{O2}$  is transmitted to  $v_{O3}$ , which force  $v_{O4}$  to 5V. Thus the input information is transmitted to output during one clock cycle.



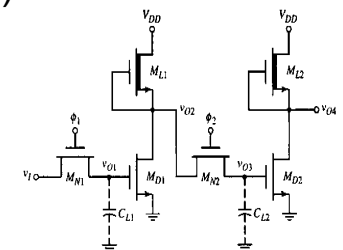
## Dynamic Shift Registers at Various Time

- Consider when  $t=t_4$ ,  $v_i=0$ , and  $\phi_1=5V$ , so  $V_{O1}=0$  and  $V_{O2}=5V$ .  $V_{O3}$  and  $V_{O4}$  depend on previous history
- At  $t=t_5$ ,  $\phi_2=5V$ ,  $v_{O3}$  charges to  $V_{DD} - V_{TN}=4V$  and  $V_{O4}$  goes low.
- Thus logic 0 is shifted (transmitted) from input to output.
- Also note that  $v_{O3}$  and  $v_{O4}$  are depend on **previous history** of their inputs instead of current inputs (they are having **memory**).



## NMOS shift register is also dynamic (why?)

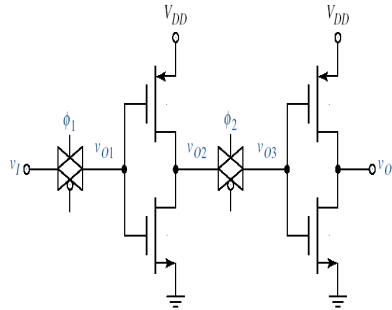
- The output charged capacitor does not remain constant with time because it is discharge through the transmission gate transistor.
- In order to prevent logic errors, the clock signal period  $T$  must be small compared to effective **RC** discharge time constant.



For example at  $t = t_2$ ,  $V_{O1}=4V$ ,  $\phi_1=0$  and  $M_{N1}$  is cutoff.  $V_{O1}$  will start to decay and  $V_{O2}$  will begin to increase.

## CMOS Dynamic Shift Registers

- The operation of the CMOS shift register is similar to the NMOS register except for the **voltage levels**.
- For example, when  $V_I = \phi_1 = V_{DD}$ . Then  $v_{O1} = V_{DD}$  and  $v_{O2} = 0$ . when  $\phi_2$  goes high, then  $v_{O3}$  switch to zero,  $v_{O4} = V_{DD}$ .
- Thus input signal is shifted to the output during one clock cycle.

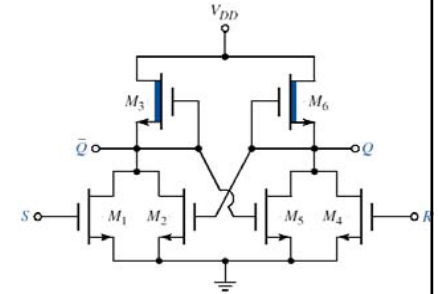


## NMOS R-S Flip Flop

- Flip-flops are **bistable** circuits usually formed by **cross-coupling two NOR gates**. The output of the two NOR circuits are connected back to the inputs of the opposite NOR gates.

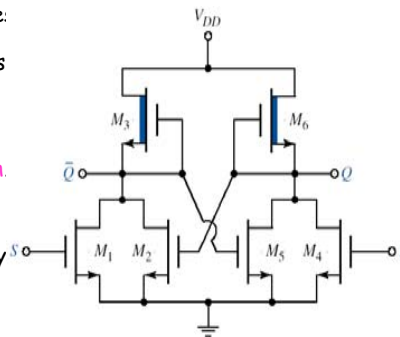
When  $S = \text{logic 1}$  and  $R = \text{logic 0}$ ,  $\bar{Q} = \text{logic 0}$  and  $Q = \text{logic 1} = V_{DD}$ . Transistor  $M_2$  is then also biased in conducting state.

If  $S$  returns to logic 0, nothing in the circuit can force a change and flip flop stores the previous logic states, although  $M_1$  turned off (but  $M_2$  remains turned on).



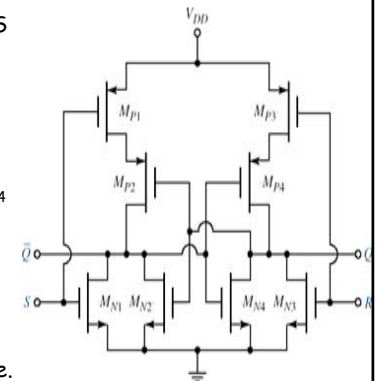
## NMOS R-S Flip Flop (cont.)

- When  $R = \text{logic 1}$  and  $S = \text{logic 0}$
- Then  $M_4$  turn on so output goes low. With  $S = Q = \text{Logic 0}$ , both  $M_1$  and  $M_2$  are cutoff and goes high. The flip-flop is now in reset state.
- If both  $S$  and  $R$  inputs go high. Then both outputs  $Q$  and  $\bar{Q}$  would go low, which implies that output is not complementary. This condition is **forbidden or nonallowed condition**.



## CMOS R-S Flip-Flop

- The operation sequence of CMOS R-S flip flop is same as NMOS.
- For example: If  $S = \text{logic 1}$  and  $R = \text{logic 0}$ , then  $M_{N1}$  is turned on,  $M_{P1}$  is cut off, and goes low.
- With  $\bar{Q} = R = \text{logic 0}$ , then both  $M_{N3}$  and  $M_{N4}$  are cut off, both  $M_{P3}$  and  $M_{P4}$  are biased in a conducting state so that the output  $Q$  goes high.
- With  $Q = \text{logic 1}$ ,  $M_{N2}$  is biased on,  $M_{P2}$  is biased off, and the flip-flop is in a **set condition**.
- When  $S$  goes low,  $M_{N1}$  turns off, but  $M_{N2}$  remains conducting, so the state of the flip-flop does not change.



## CMOS R-S Flip-Flop (cont.)

- When  $S = \text{logic 0}$  and  $R = \text{logic 1}$ , then output  $Q$  is forced low, output  $\bar{Q}$  goes high, and the flip-flop is in a **reset condition**.
- Again, a **logic 1 at both  $S$  and  $R$**  is considered to be a forbidden or a nonallowed condition, since the resulting outputs are not complementary.

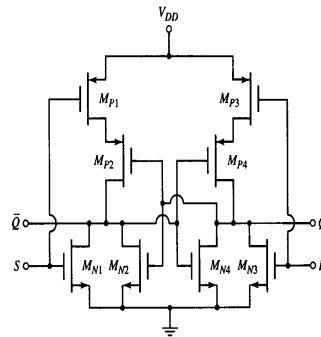


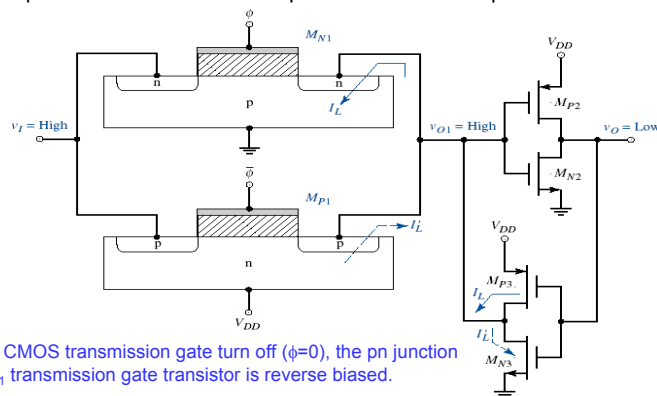
Figure 16.72 CMOS R-S flip-flop

## Static vs Dynamic Storage

- Static storage**
  - preserve state as long as the power is on
  - have positive feedback (**regeneration**) with an internal connection between the output and the input
  - useful when updates are infrequent (clock gating)
- Dynamic storage**
  - store state on parasitic capacitors
  - only hold state for short periods of time (milliseconds)
  - require periodic refresh
  - usually simpler, so higher speed and lower **power**

## Static D-type Flip-Flop

- A D-type flip-flop is used to provide a delay. The logic bit on the D input is transferred to the output at the next clock pulse.



When the CMOS transmission gate turn off ( $\phi=0$ ), the pn junction in the  $M_{N1}$  transmission gate transistor is reverse biased.

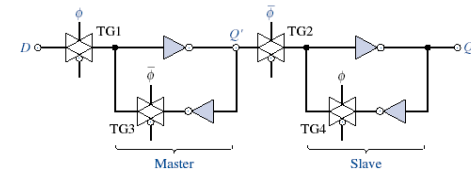


Figure 16.74 CMOS master-slave D flip-flop

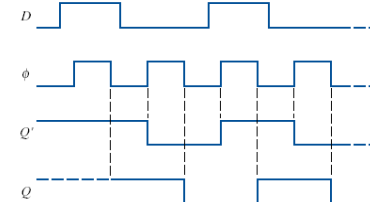


Figure 16.75 D flip-flop signals at various times